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Designing an Experimental Prototype for the Teaching of Conics (Ellipsis) based on the Law of Light Reflection

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Abstract

This paper aims at describing an experimental prototype to favor the comprehension of theoretical concepts related to the section of conic in Geometric Optics courses or subjects such as Mathematics and Analytical Geometry. The experimental prototype was devised for lab practices intended to demonstrate conics by means of the law of light reflection and its application in the social context. The prototype was thought of as a solution to cope with expensive optical kits. It is an alternative for educational institutions to overcome financial difficulties. Instead the prototype may be constructed at the institutions using components available at the market. It covers a wide range of optical classroom teaching conic activities and has the flexibility for incorporating new ones. The use of the prototype is recommendable for Math and Physics lab activities at the preparatory school and the faculties of Nuevo León Autonomous University.

Keywords: Educational experiment, laboratory experiment, geometry, optical geometry

Introduction

The laws of reflection and refraction of light of Geometric optics has been the theoretical foundations for devising and constructing optical tools such as binocular, cameras, and telescopes. These refracting or reflecting tools are based on conic components taking advantage of the lack of spherical aberration of aspheric surfaces that allows the construction of compact telescope of high resolution. Many scholars have conducted academic research to improve the comprehension of Physics and Mathematics courses both at high school and university levels. They pursued the description of pedagogical and didactic strategies facilitating the process. It seems that the royal road to success is the relation of theory and practice, based on the assumption that discovering practical applications of scientific notions will promote sound learning.

1. Designing the experimental optical prototype

The proposed prototype (Ruiz, 2017) describes a portable modular device for the practical study of conics by means of reflection. It is composed of a container, which includes a light source, converging spherical lenses and converging lenses of cylindrical type; a set of diaphragms including a diaphragm with image, or grids (going from three to ten slits). All components are easily stackable to store them compactly (figure 1).

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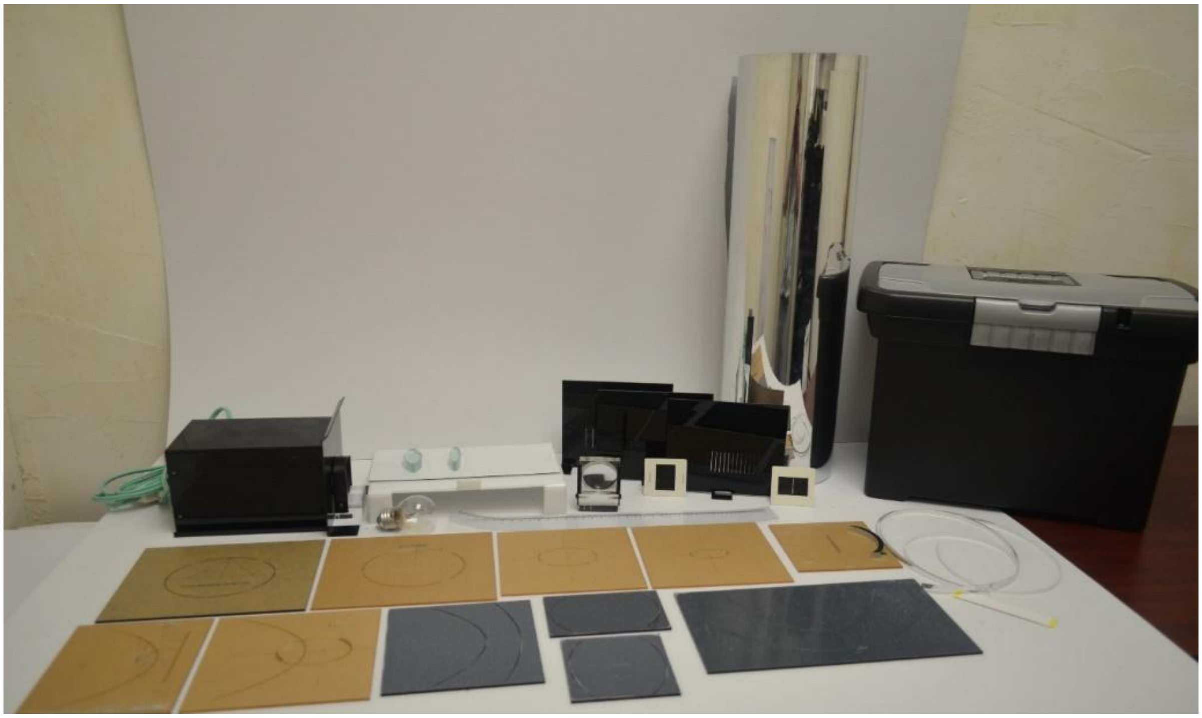


Figure 1: Geometric optics prototype for conics

An instruction manual was written to explain the potential of the geometric optics prototype for conics. A wide range of demonstrations are possible, going from the most basic practices with light, to configurations of lenses to form composite optical devices, where teachers and students will be exposed to simple technology such as spotlights, lenses and simple mechanisms that will provide quantitative measurements of the conics (circles, parabolas, ellipses and, hyperbolas).

2. Related mathematics notions

The conics were first described by Menecmo (350 BC), but the mathematician Apollonius of Prague (260 - 190 BC) was the first to study conical curves in detail identifying their properties. One of the most important and useful properties that he discovered was the property of reflection (Vera, 1970). Conics are usually defined as a set of points intersecting a plane with a cone of revolution of two branches. If the plane is perpendicular to the axis of the cone, the intersection is a circumference or a point, depending on whether it intersects a branch or passes through the vertex. If the plane is not perpendicular to the axis but intersects every generatrix, the intersection is an ellipse. If the plane is parallel to one generatrix and cuts all others, the intersection is a parabola. If the plane cuts two branches of the cone and nothing passes through the vertex, the intersection is a hyperbola. If the plane passes through the vertex, the intersection is a point, two lines that are cut, or a single line (figure 2).

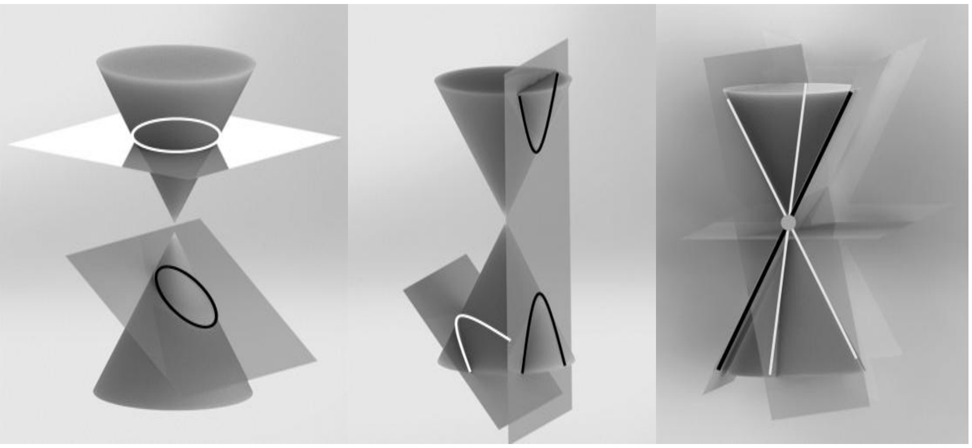


Figure 2: Perspective of conic sections

An ellipse (figure 3) is the locus of a point that moves in a plane in such a way that the sum of its distances to two fixed points of that plane is a larger constant than the distance between the two points called foci of the ellipse, the definition of an ellipse excludes the case in which the moving point is on the segment that joins the foci (Lehmann, 1989, p. 174).

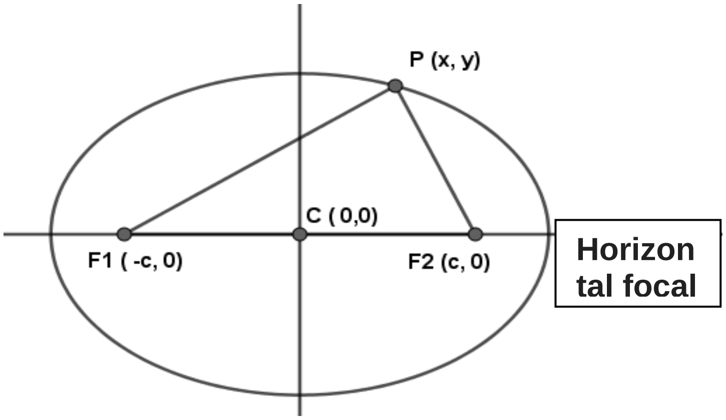
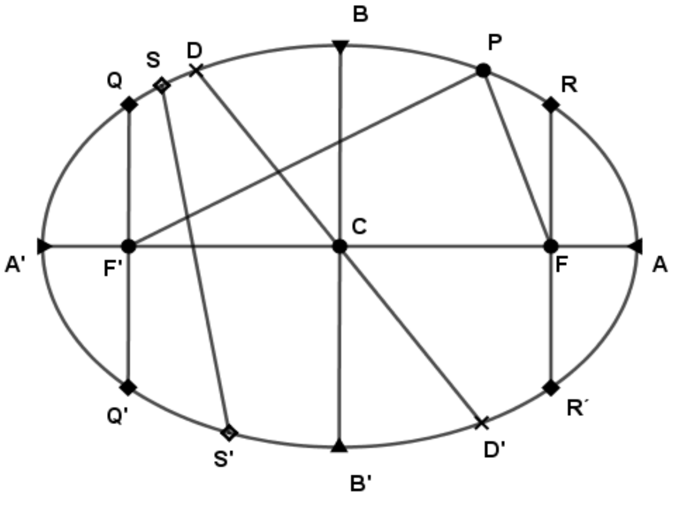


Figure 3: The representation of the ellipse, a point, and its foci.

The foci of the ellipse are two equidistant points from the center, *F1,* and *F2* in the larger axis. The sum of the distances from any point *P* of the ellipse to the two foci is constant, and equal to the length of the largest diameter (d (P, F1) + d (P, F2) = 2a) where 2a is the measure of the larger axis of the ellipse. Figure 4 illustrates the elements of the ellipse.



C – Center of the ellipse
A y A’ - The vertex of the semi-mayor axis
AA’ – Major axis
B y B’ – The vertex of the semi-minor axis
BB’– Minor axis
F y F’ – they are the spotlights
PF y PF’ – radius vector
RR’ – Straight side
QQ’ – Straight side
DD’ - Line that goes through the center called Guideline

Figure 4: Elements of an ellipse.

According to the definition of the ellipse, it can be verified that the segment of the line joining the focus F with the vertex of the smaller semi-axis has the magnitude of (a) that agrees with the magnitude of the larger semi-axis; the length from the center of the ellipse to the focus can also be obtained by the Pythagorean theorem as seen in figure 5.

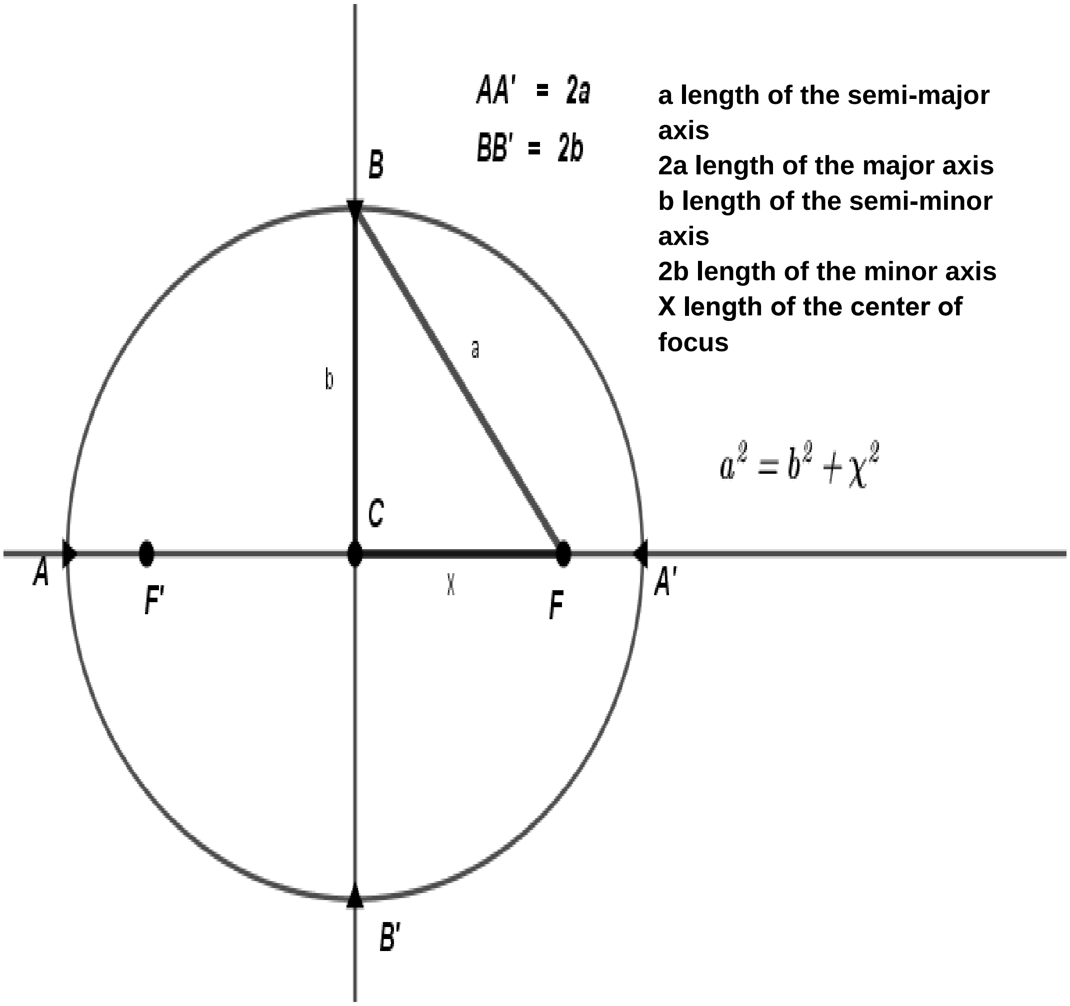


Figure5: Ellipse defining elements

History shows that the application of the properties of conics has been very useful either in the study of astronomy or in the construction of churches, galleries and science museums where the property of the reflection of the light is used in the same way that sound or electromagnetic waves are reflected. This explains that a person standing in one of the foci of the ellipse can hear the sound emitted from the other focus. Likewise, the revolution surface theory explains that if an ellipse is rotated around its larger axis on a surface it generates an ellipsoid; if the inner surface of the ellipse is represented as a reflecting surface such as a mirror, light that making an impact on one of the bulbs will be reflected towards the other focus. Such behavior will be shown with the experimental equipment.

2.1. Description of some physical concepts of Geometric Optics

In physics, Geometric Optics is based on the concepts of lightning, the refractive index of materials (glass) and the path or optical path of a beam through one or more materials. Cornejo and Urcid (2005, p. 5) explain that geometric optics analyzes physical problems by means of the laws of reflection and refraction of light, in one or more reflecting or refracting surfaces. Knowledge of physical concepts is helpful for explaining the conic section. The laws of reflection and refraction and convergent lenses are defined below. As explained by Hecht and Zajac (1974), if a light beam propagating through a homogeneous medium *N1* impinges on the surface of a second homogeneous medium *N2*, part of the light is reflected and part enters as a refracted ray in the second medium. The plane of incidence is defined as the plane formed by the incident beam, the reflected beam, the refracted beam and the normal line (that is, the line perpendicular to the surface of the medium) at the point of incidence (figure 6).

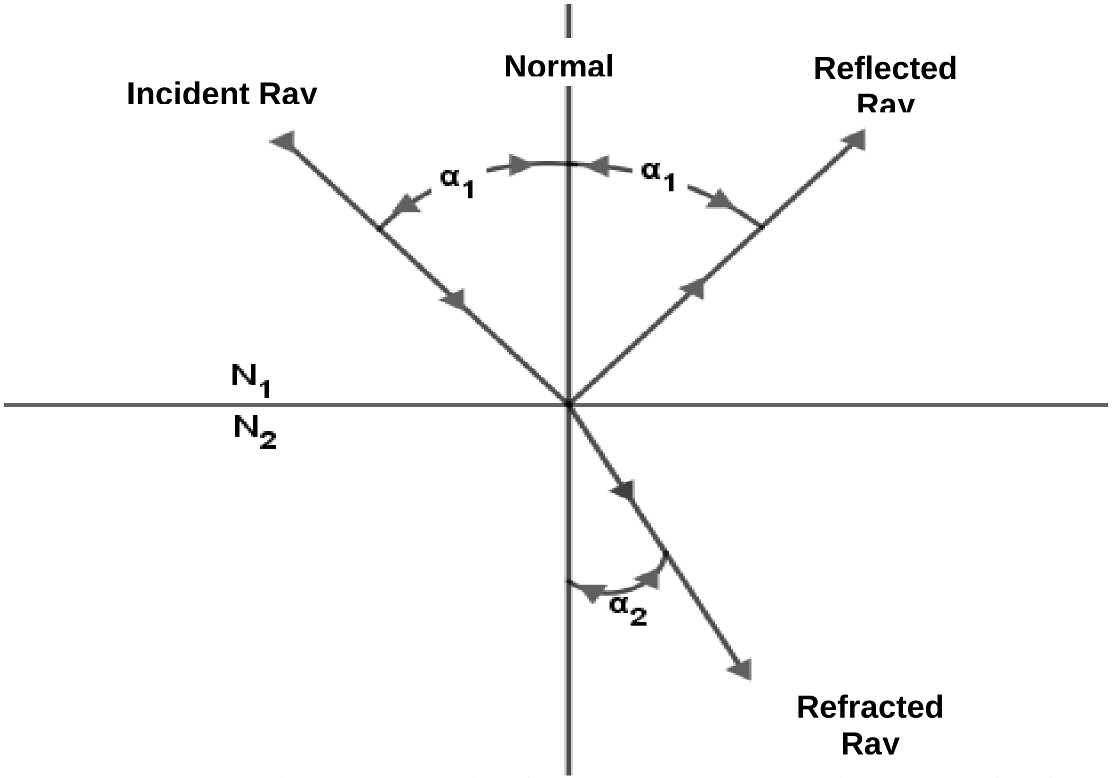


Figure 6: Relationship between an incident beam on a flat surface and the reflected and refracted beams. According to the law of Reflection, 1) the incident, the normal and the reflected beams are in the same plane, and 2) the angle of incidence *a1*, is equal to the reflection angle $\alpha 1$. Snell's Law explains that the product of the refractive index of the first medium and the alpha of the angle of incidence of a beam is equal to the product of the refractive index of the second medium and the alpha of the refraction angle. Thus, The following equation results: $N1 \sin \alpha_1 = N2 \sin \alpha_2$ Notice that the laws explaining optical systems (lenses, mirrors and optical instruments) are deduced from the Laws of Reflection and Refraction and using plane geometry formulas. Young and Freeman (2009) call "optical system" to the combination of spherical and aspheric surfaces possibly constructed with materials of different refractive index. These surfaces can be refractive or reflective. The lenses are transparent objects (usually glass) limited by two surfaces that can be of different curvature. Convergent lenses (positive lenses) are thicker at the central part and narrower at their edges, so called because light beams that strike parallel to the main axis of the lens, cross through a point called focal point (image focus) (figure 7).

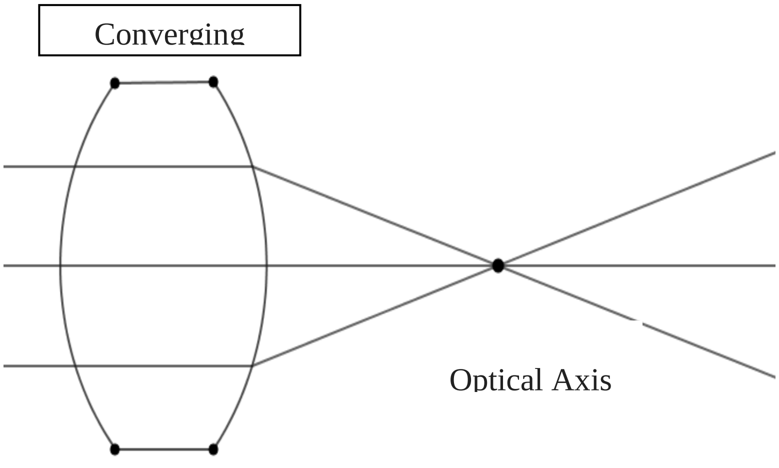


Figure 7: Convergent lenses

3. Experimental demonstration

The proposed optical prototype may be for proving experimentally the course of light beams, the laws of reflection and refraction, the focal lengths of converging lenses, the experimental arrangement of reflecting telescopes, and the like. The instruction manual provides a full description of the following experimental practices.

3.1. Parallel beams demonstration

The experimental equipment and its respective instructions allow several experimental arrangements. The illustration below shows an experimental arrangement in which three parallel beams are obtained.

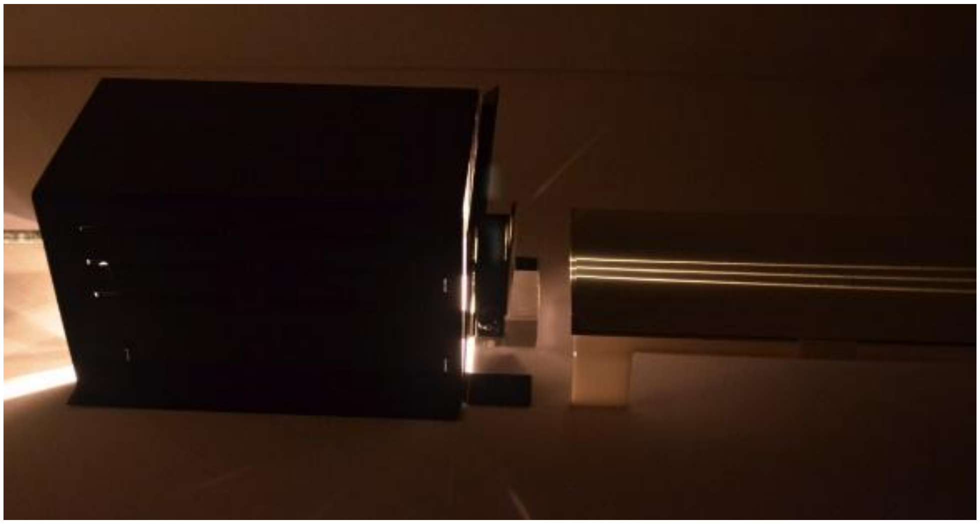


Figure 8: Experimental arrangement to obtain parallel light beams on a collecting table.

Other variants are possible. For example, At the experimental arrangement of three parallel rays of figure 8, interpose one of the converging lenses of the equipment and observe how the three beams intersect a point called focal distance of the lens, which agrees with the explained theory of geometric optics for converging lenses (figure 9).

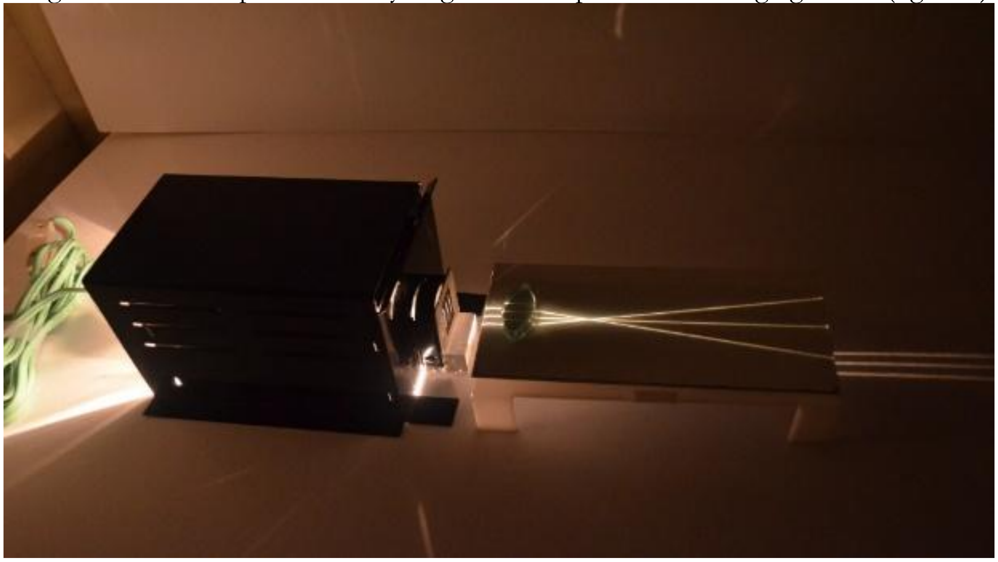


Figure 9: Parallel beams crossing or converging at the focus of the biconvex lens.

3.2. Law of reflection demonstration

An experimental arrangement to illustrate the law of reflection is also possible. The grid of three slits is replaced by one of a single line and a elliptical mirror (figure 10) is used instead of the converging lens of the preceding practice.

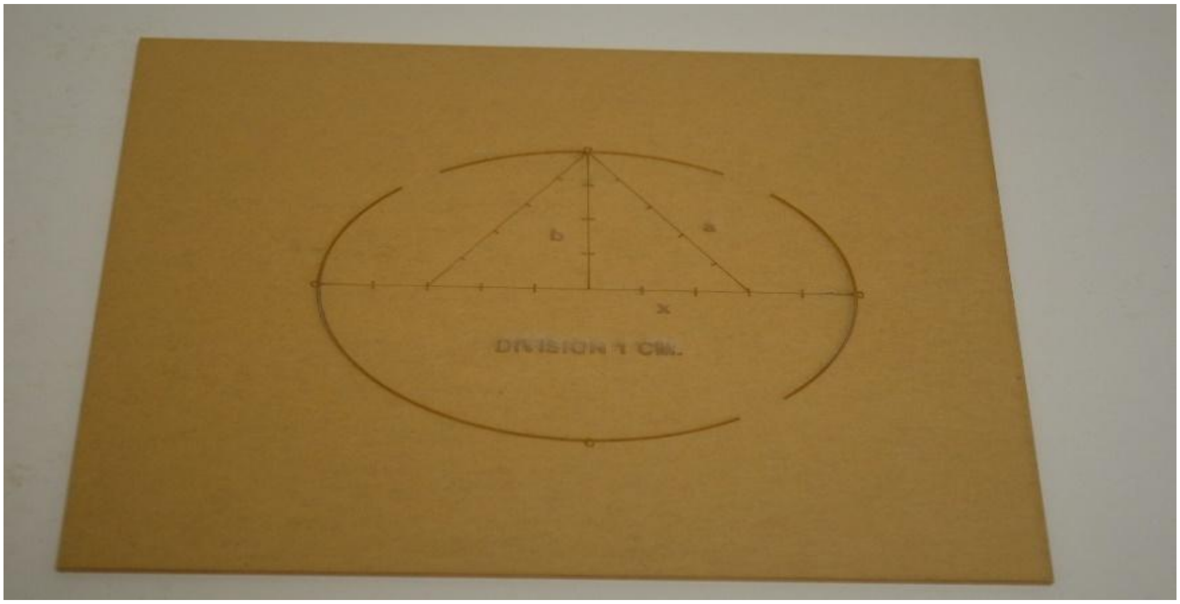


Figure 10: Elliptical components of a larger semi-axis of 10 cm and a shorter semi axis of 8 cm

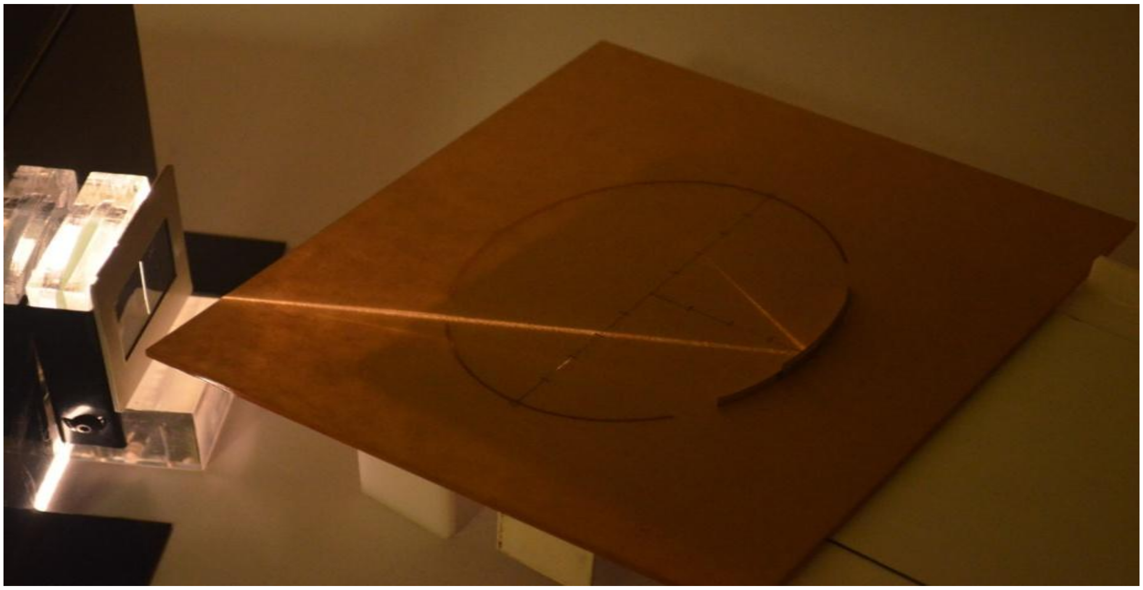


Figure 11: Reflection of a ray of light for an elliptical surface

As illustrated at figure 11, in that experimental arrangement the beam is reflected with an angle similar to the normal. There is symmetry of the incident beam to the reflected beam as the normal line coincides with the ellipses smaller semi-axis. Moreover, this arrangement may be geometrically verified; it forms an isosceles triangle, that is to say two equal sides and one unequal that coincides with the larger axis. According to the figure, these sides have a magnitude (a) and their sum (2 a) determines the magnitude of the ellipses larger axis, going from vertex to vertex.

3.3. Localizing the foci of an ellipse semi-axis

Now a three-slit grid is used, then the elliptical components of a larger semi-axis of 10cm, a shorter semi-axis of 8 cm, and a focus 3cm long (figure 10). Place the converging lens in such a way that the beams, when converging, cross the lens a point coinciding with one of the foci of the ellipse. One of these beams is intentionally adjusted to strike the vertex. According to the definition of the ellipses, the reflected beams must cross the second focus of the ellipse as shown in figure 12.

Figure 12: Component of the ellipse with the beam passing through its foci

Once this activity is carried out, the distance between one focus to the vertex is measured, then the same procedure is completed for the second focus. The aim is to prove that the distance of the vertex to both foci is the same, and that its sum is the value of 2 *a* larger than the distance of focus *F1* to focus *F2*, but equal to the distance from vertex to vertex of the larger axis which agrees with the definition of the ellipse, so that definition has been demonstrated by the laws of light reflection. Consider the experimental arrangement illustrated in figure 12. Measure the distance of one of the beams from one of the ellipse focus to the reflecting surface, and again from it to the other focus (figure 13). Then proceed in the same way with the other two beams. You will verify that its magnitudes are equal to a constant, so the definition of the ellipse indicates that this constant will be the value of 2 *a* from vertex to vertex distance of the larger semi-axis.

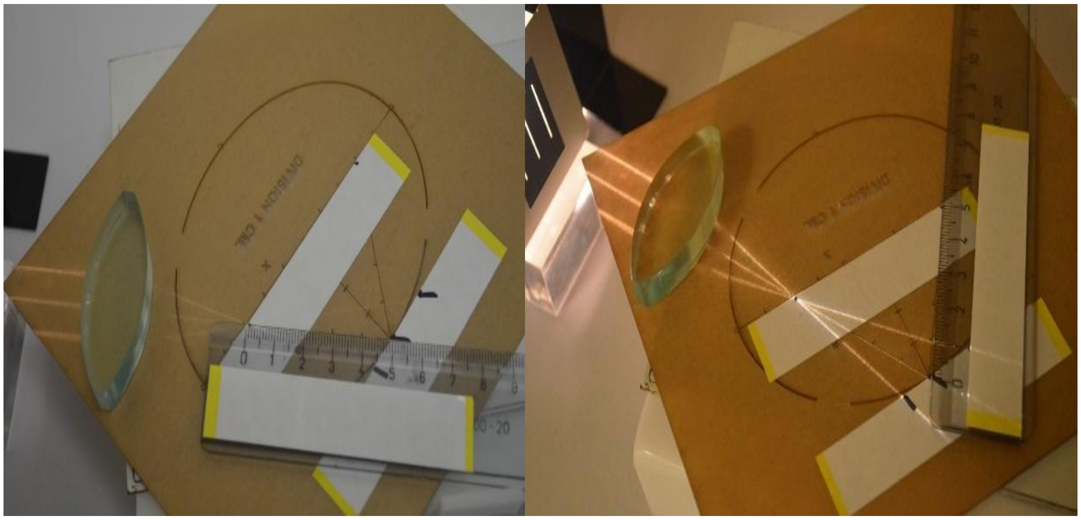


Figure 13: Measurement of a beam from the focus to the vertex of the smaller semi-axis.

According to the definition of the ellipse, any light beam that passes through the focus when reflected on any of the surfaces of the ellipse will pass through the other focus as seen in the following figure 14. In the same way, beams crossing through one of the foci of the ellipse once reflected will pass through the other focus. If we assume that it continues to reflect, it will go on successively through one focus and another. Here, only two reflections are shown that agree with what has been said (figure 15).

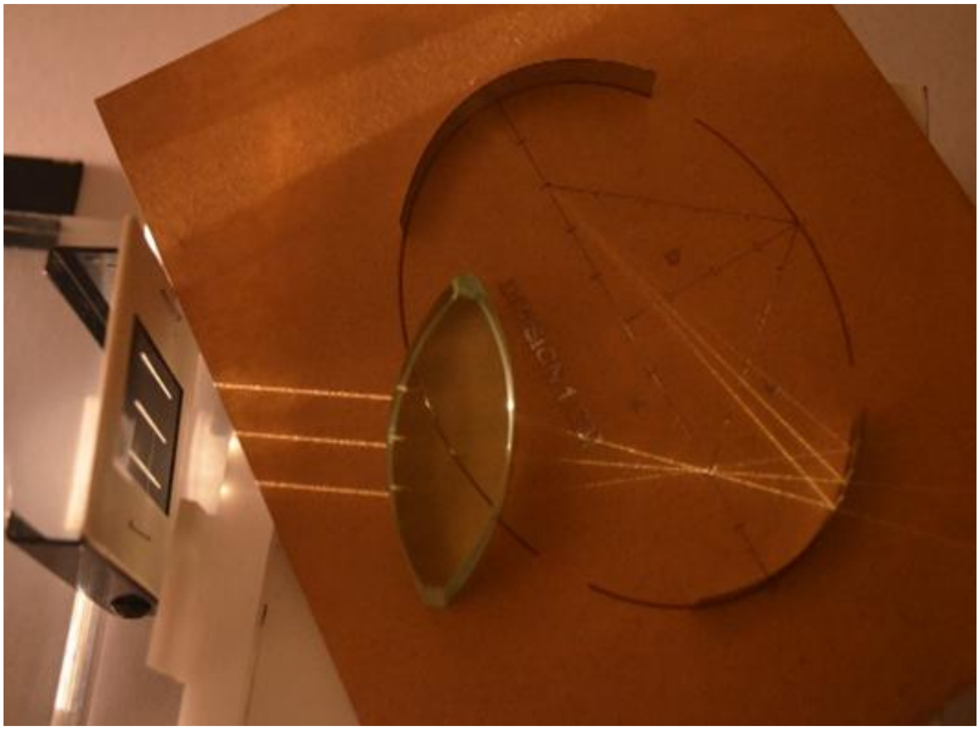


Figure 14: Reflection of light in different sections of the ellipse.

Figure15: Multiple reflections of light passing through ellipse its foci.

Remembering that non-spherical surfaces have sometimes been used as light projectors where the light source is in one of the spotlights, here we simulate the light source using the beams emerging from the lens to one of the focus as if the source was in the focus and the reflected beams were parallel projected (figure 16).

Figure 16: Ellipse projecting parallels beams.

4. Application of the ellipse

According to the geometric optics, spherical surfaces have sphericity aberration, that is to say, that the beams striking parallel to the axis of the surface do not cross at a point which determines that each beam has a different focal distance (figure 17). This effect does not happen on conical surfaces (ellipse, parabola, hyperbola), for example on the surface of the parabola the parallel beams to the surface axis when reflected crossed by a point as shown in figure 17, that is, they lack spherical aberration. Designers of optical instruments possibly take advantage of the characteristic feature of conics for telescopes design. This basic principle was used by Gregory for the design of his telescope.

a) Circle b) Parabola

Figure 17: Images of the circle and parabola reflecting beams striking the surface.

In 1663, James Gregory, famous Scottish mathematician, published a book entitled *Promica Optica*, describing a system shown in figure 18 and used for devising a telescope. He used a primary parabolic mirror and a secondary elliptical mirror to avoid spherical aberration. The focus of the parabola coincides with one of the foci of the ellipse and the image is formed in the other focus of the ellipse, where an eyepiece is placed to observe the image.

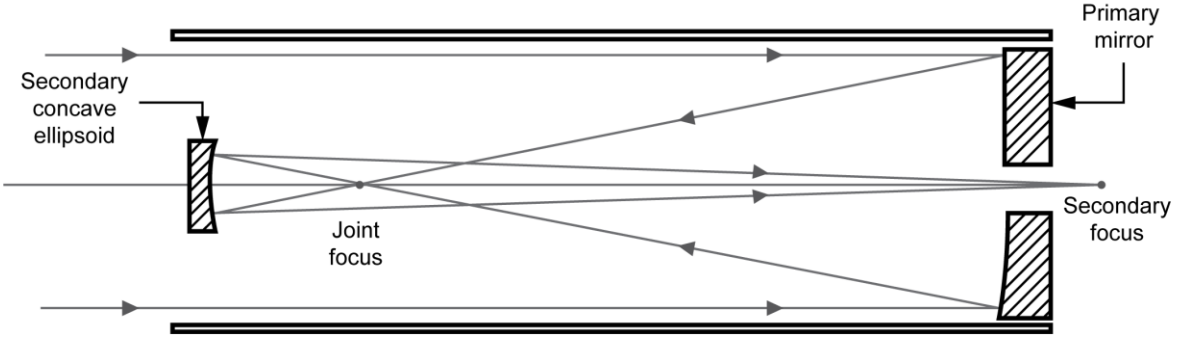


Figure 18: Diagram of a Gregorian telescope from 1873.

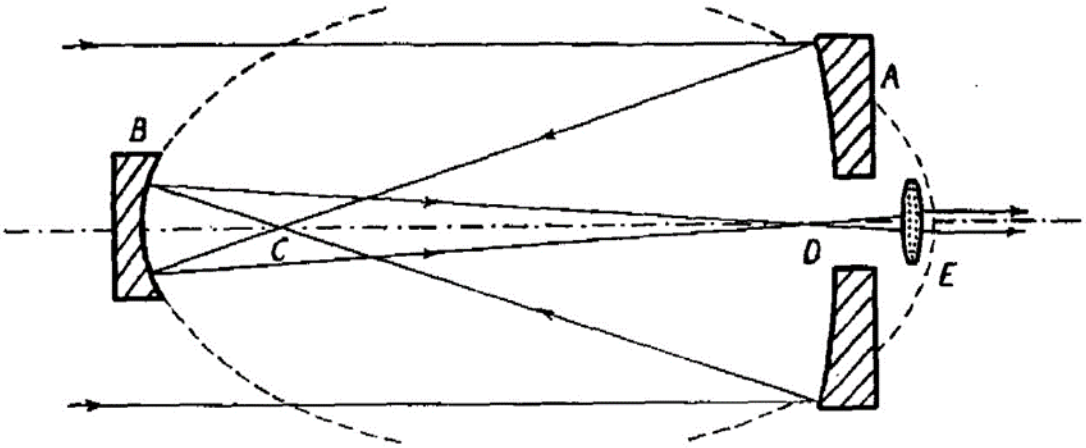


Fig. 33—The Gregorian reflector

A is the primary mirror, a concave paraboloid centrally perforated. *B* is the secondary concave ellipsoid. Light from a star is sent towards *C*, the joint focus of *A* and *B*, and reflected to *D*, the other focus of the ellipsoid. The image at *D* is then observed with an eyepiece, shown as a single equi-biconvex lens *E*.

Figure 18: Image taken from the book The History of telescope.

The optical prototype herein proposed may also be helpful for explaining Gregory's telescope functioning. For that purpose, the professor did the necessary experimental arrangements to make the focus of the parabola coincides with one of the foci of the ellipse (figure 19).

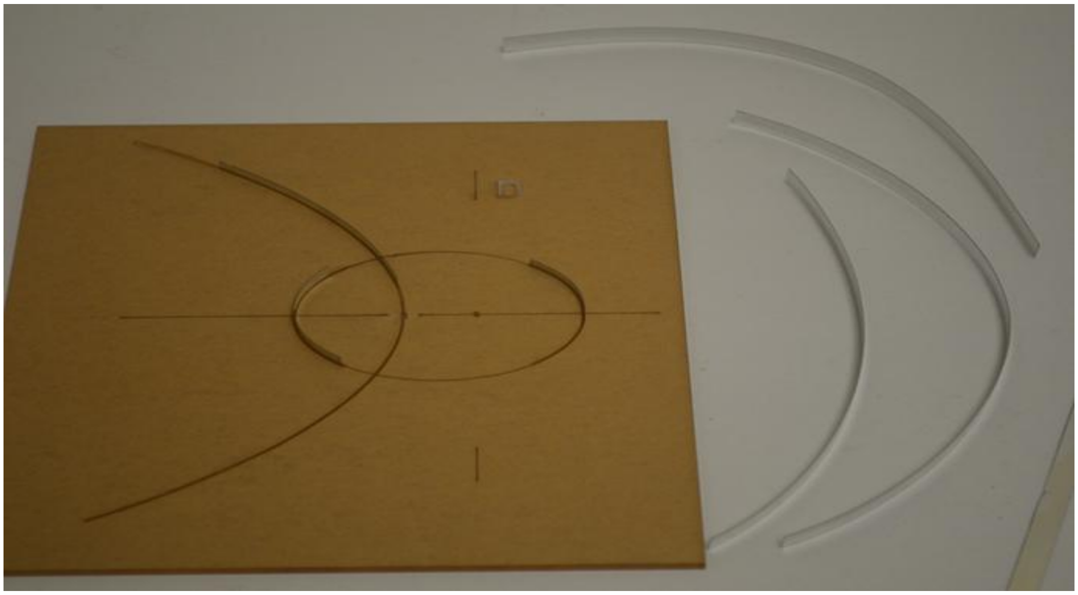


Figure 19: Parabola and ellipse with a common focus

The experimental arrangement is carried out with the help of the conical section optical equipment instruction manual. A beam of light is adjusted to go parallel to the axis of the parabola and ellipse to strike the surface of the semitransparent mirror of the section of the parabola, part of the light is reflected and part is transmitted, the reflected beam passes through the focus of the parabola that coincides with the focus of the ellipse. According to the preceding demonstration, any beam passing through the focus will be reflected on the surface of the ellipse and will cross by the other focus of the ellipse; here it is observed that the beam transmitted from the parabola is parallel to the axis of the parabola and ellipse. The intention is to make the beam to strike upon the straight side of the ellipse, by definition it will cross through the focus of the ellipse (in this experimental arrangement the reflected and transmitted beams intersects at a common point that represents the focus of the ellipse (figure 20). Thus we have demonstrated the basic concept behind the functioning of Gregory's telescope based on the laws of reflection.

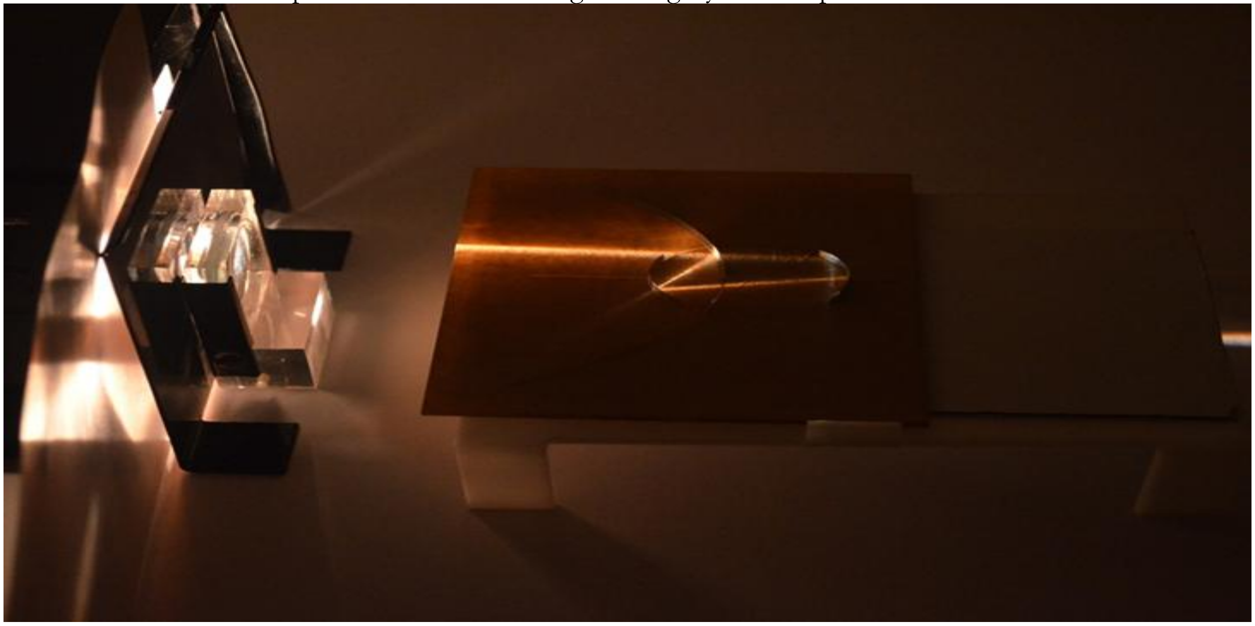


Figure 20: Simulating Gregory telescope functioning.

This experimental arrangement allows verifying that the parabola focal distance and the ellipses focal distance are congruent with the telescope design proposed by Gregory, see figure 21.

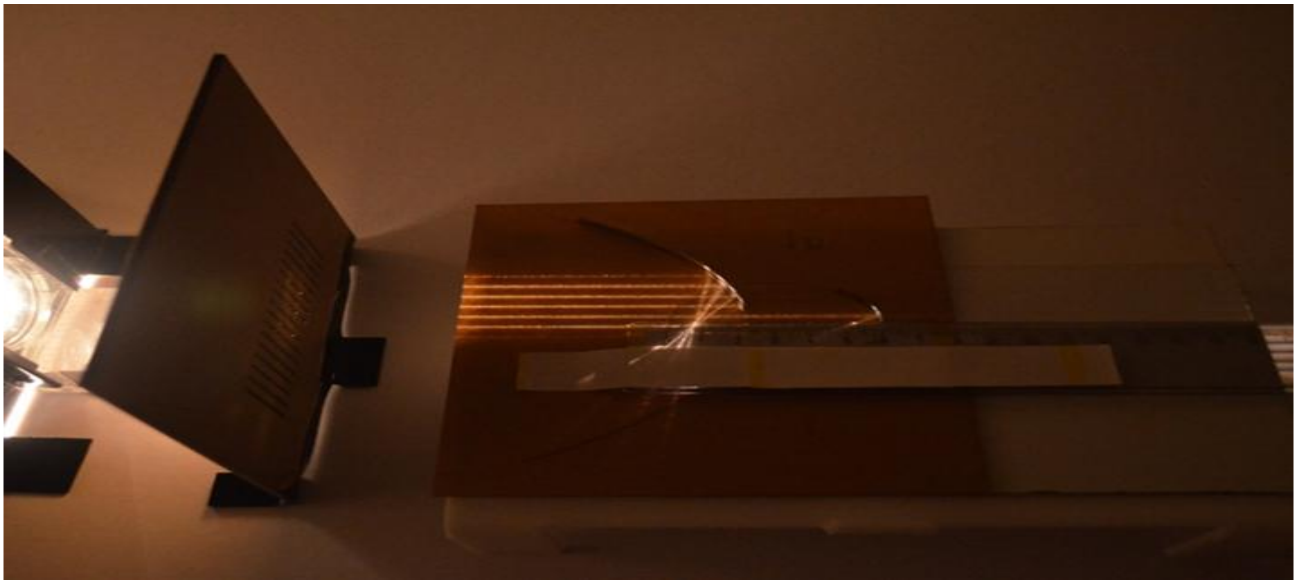


Figure 21: Measuring the focus of the parabola and ellipse

In this way, it is possible to explain the theory of the conic section of the ellipse associating it to laws of light reflection. Mathematic abstract notions are experimentally illustrating by using the equipment designed.

Conclusions

The proposed way of explaining the concept of conics can be an alternative way, which allows instructing teachers to relate theory to practice and to enhance the understanding of mathematical concepts in students.

The immediate plan is to generalize the introduction of the proposed optical prototype at the Autonomous University of Nuevo León, México and to get good results in the development of students' competencies, and achieving a compressive education by related theory to practice.

References

Cornejo, A., & Urcid, G. (2005). *Óptica Geométrica* (Vol. I). México: Instituto Nacional de Astrofísica, Óptica y Electrónica.

Hecht, E., & Zajac, A. (1974). *Optics* (4th ed.). Boston: Addison Wesley. Retrieved April 17, 2018, from https://www.fing.edu.uy/sites/default/files/cursos/2013/anexos/6643/OPTICA.pdf

Lehmann, C. H. (1989). *Geometría Analítica.* México, D. F.: Limusa.

Malacara, D. H., & Malacara, J. M. (2015). *Telescopios y estrellas* (4th ed.). México, D. F.: Fondo de Cultura Económica.

Ruiz, J. C. (2017). *Diseño de un prototipo óptico para el estudio de las cónicas.* Nuevo León, México: Facultad de Ciencias Físico Matemáticas, Universidad Autónoma de Nuevo León .

Vera, A. (1970). *Introducción y notas.*Retrieved April 17, 2017, from http://www.monografias.com/trabajos82/secciones-conicas-aplicaciones/secciones-conicas-aplicaciones.shtml

Young, F., & Sears, Z. (2009). *Física Universitaria con Física Moderna* (12th ed., Vol. II). Retrieved April 17, 2018, from www.pearsoneducacion.net