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A Design of Teacher Principles for the Development, Diffusion, and Appropriation of Students' Mathematical Knowledge

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Abstract

This study documents and describes results of a classroom design study that investigated middle school mathematics classrooms where students were introduced to new mathematical tools, tool-related practices, concepts, facts, and problem solving strategies through the sharing of student-initiated ideas throughout the classroom. A resulting product of this design study is a set of multi-revised principles for mathematics teachers to use to promote the development, diffusion, and exchange of innovative mathematical knowledge in classrooms where students work collaboratively on problem-based mathematical tasks. The implementation principles were designed using the established research framework of diffusion theory (da Ponte, 2013; Rogers, 2003) and are intended to guide teachers in modifying the classroom environment to promote the sharing, spread, and exchange of mathematical ideas, facts, concepts, problem solving strategies, and tool usages.. Utilizing a pattern coding method to analyze student interaction and work samples, the principles underwent four testing iterations in a design experiment. The final set of constructed principles provide implications for middle school mathematics teacher education.

Keywords: diffusion of mathematical knowledge; middle school

1. Introduction

Much of the reform in mathematics education advocates teachers supporting collaborative learning and approaches that require students to construct their own mathematical ideas . What forms of knowledge and experiences do mathematics students need to develop into such a student? In this study, the researcher focused attention on studying a middle school mathematics classrooms where students were introduced to new mathematical tools, tool-related practices, concepts, facts, and problem solving strategies through the spread of student-initiated ideas throughout the classroom. Identifying, describing, and categorizing events and conditions that foster such circulation of ideas led to the design of a set of multi-revised implementation principles for teachers to use as a guide for constructing and facilitating a classroom environment where information and ideas are exchanged among classmates. These designed implementation principles, defined as general guidelines that establish a basis for reasoning, suggest a distinctive method and describe a mode of action to guide teachers in modifying the classroom environment to promote the sharing, spread, and exchange of mathematical ideas, facts, concepts, problem solving strategies, and tool usages. One of the purposes of developing teacher principles is to clarify means for engaging students in the processes of knowledge advancement. The challenge is not simply to provide opportunities for collaboration but to design classroom environments so that knowledge is shared. The effectiveness of the principles was tested using the premise of a conventional type of educational design research, known as design experiments (Kelly, Lesh & Baek, 2008; van den Akker et al., 2006), that is modeled after design research in applied fields such as engineering. The principles underwent four testing iterations in which they were revised and tested until a polished set was constructed.

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Research results from this study highlight factors that influence the development, diffusion, and appropriation of mathematical knowledge among middle school students who work collaboratively on thought revealing mathematical activities known as model-eliciting activities (Chamberlain & Moon, 2008; Lesh, Hover, Hole, Kelly, & Post, 2000). As used in this study, mathematical knowledge is a broad term referring to mathematical ideas, facts, concepts, problem solving strategies, tool related practices, etc. Such a broad term is used in order to capture more than the notion of the sharing and exchange of ideas but also the diffusion of other common constructs and practices. Diffusion, in contrast to sharing or spreading, is a representation of an element breaking apart before it travels (da Ponte, 2013; Rogers, 2003). Everett Roger's theory for the diffusion of innovations provides a view on the factors that influence the change or lack of change on the practice of members of an organization (i.e. a classroom). It is a theory that is often applied to phenomena of organizational change both within and outside of education. Diffusion theory highlights characteristics of innovations that influence participants' (i.e. students) decisions to accept or reject the innovation (i.e. mathematical idea). In this study, the term diffusion refers to the movement or circulation of students' innovations, specifically their mathematical ideas or other knowledge. In this movement, students' ideas divide and students connect different ideas to form new ones. These ideas are restructured, built on, and interpreted differently as they circulate and are shared throughout a classroom setting. However, students do not simply acquire or are exposed to ideas passed around the classroom. This process, referred to as appropriation (Windschitl, 2001), is more of a transaction where ideas are interpreted, negotiated, defended, and adapted. In the process of appropriating ideas or knowledge, research shows that students incorporate ideas from others into the development of their own models and use these ideas in other aspects of the problem as well as other problems

2. Background on Model-Eliciting Activities

Model-eliciting activities (MEAs) are a crucial component of this design study. MEAs were developed by mathematics education researchers to better understand and encourage problem solving (Lesh, 2002). MEAs are designed to encourage students to build mathematical models in order to solve complex problems, as well as provide a means for educators to better understand students' thinking. Additionally, MEAs have been adapted for varied grade levels and other disciplines such as engineering (Diefes-Dux et al, 2006) and gifted education (Chamberlain and Moon, 2008). In a typical mathematics classroom, there may be important but unobservable problem-solving processes (i.e. mental thinking) transpiring. The use of model-eliciting activities enables a student's thinking to be externalized and documented. Each activity requires students to work in small groups of three to four students to mathematically interpret a situation and provide a logical solution based on their interpretation. Instead of being a one word, one number, or "show your work" answer, students' solutions are models that reveal how students are thinking about a given situation. Students' models/solutions range from being a mathematical description, procedure, or method designed for the purpose of making a decision for a realistic client from the problem statement. The students' descriptions, explanations, and constructions reveal how they are thinking about the mathematical situation by disclosing how these situations are quantified, organized, coordinated, and interpreted. In this way, MEAs are replications of real life problem solving situations that elicit students' mathematical understandings and allow them to document their own thinking about a situation that can be mathematically modeled. For example, The Paper Airplane MEA requires middle school students to read a designed newspaper article that describes how to make a variety of different types of paper airplanes. Working in teams, students are then given a data sheet with results produced by another group of students showing three different flight paths. For each flight path, three measurements are recorded: total distance flown, distance from target, and time in flight. The students are then required to write a letter to students in another class describing how such data can be used to assess paper airplanes for three kinds of flight characteristics: best boomerang, best floater, and best overall.

2.1 Model-Eliciting Activities Observed in Study

Implementation of the following six MEAs were observed for this study.

i. *Bigfoot* MEA required students to develop a model for determining the height of a person given only the person's footprint. Mathematical concepts inherent in the problem include proportional reasoning and linear growth.
ii. *Soccer Ball* MEA required students to determine how many two inch diameter hexagons fit on an 8 ½ by 11 inch sheet of paper such that there is a minimum amount of wasted space. Mathematical concepts inherent in the problem include rotations and geometric properties of hexagons.
iii. *Diamond Roughness* MEA required students to develop a procedure for determining the roughness of a diamond using images from a high powered microscope. Mathematical concepts inherent in the problem include statistical reasoning and measurement.

iv. *Gravity Rules* MEA required students to perform several trials of measuring a person's reaction time by catching a ruler with one hand and then determining if there was a difference in response time based on hand use. Mathematical concepts inherent in the problem include estimation, data analysis, and measurement.
v. *Popcorn Delivery* MEA required students to develop optimal routes for delivering popcorn when given a street map and a window of time for making deliveries. Mathematical concepts inherent in the problem include proportionality, scaling, manipulating variable data, and finding volume of spheres.
vi. *Thanksgiving Dinner* MEA required students to develop a scheduling system to prepare and cook a list of food for Thanksgiving dinner. Mathematical concepts inherent in the problem include manipulating qualitative and quantitative data, manipulating weighted variables, scheduling, and time management.

3. Context of Study

The research design in this study is a classroom design experiment (Kelly, Lesh & Baek, 2008; van den Akker et al., 2006), where a set of principles was designed, studied, tested, and revised over iterative cycles of revision. Like design experiments in fields such as engineering, design experiments in education involve designing a physical or theoretical artifact and simultaneously studying it. In education, the artifact designed is for a learning environment, such as a classroom. Since the initial design is improved while the experiment is in progress by testing and revising it through ongoing analysis of both students' reasoning and the learning environment, both the product produced and the design of the product are important in this process. The initial design is a theoretical conjecture or physical artifact to be tested on how to support a particular form of learning.

3.1Participants and Setting

This study was conducted at an urban, public middle school in a Midwestern city in the United States, which had an enrollment of approximately 1,400 students in grades six, seven, and eight. 70% of the student population passed the state standardized exam and the ethnic distribution of the student body was 55% Caucasian, 36% African American, 5% Hispanic, 2% Asian, and 3% multiracial. This study involved three sixth grade mathematics classes that met five times a week for forty five minutes each day. There was an average of twenty students in each class observed. Because the experimentation process advanced through a number of revision cycles, the number of student and teacher participants was intentionally kept small. The two sixth grade mathematics teachers from the observed classes also participated in the study. Both teachers annually attended university affiliated professional development workshops and working groups on implementing MEAs in the middle school mathematics classroom. At the time of the study, teacher 1 (pseudonym: Emily) had three years of teaching experience and two years of experience with model-eliciting activities; whereas, teacher 2 (pseudonym: Travis) had taught for 16 years and had one year of experience with implementing MEAs in her mathematics classes. In addition to the two participating teachers, data was also collected from six groups of students (n = 24 students) in the three observed sixth grade classrooms.

3.2 Data Collection and Analysis

As it was impossible to observe entire classes of students in this context, data was collected from a random sample of twenty four students, which included two groups of students from Emily's 3rd hour mathematics class and two groups of students from Travis' 4th and 5th hour mathematics classes. None of the classes were considered remedial or honors. Students worked collaboratively in groups of four students for a total of five to seven days during the implementation of each activity. A different MEA was implemented during each testing iteration. Because the nature of MEAs is designed around group work, students were free to move around the classroom to utilize materials or other classroom resources, visit other groups, and talk with other students and the teacher. The classroom environment was student-centered. Six activities were observed over a period of 6 months, resulting in over 60 observations of 45 minute classes. There was a two week break between each testing iteration, with a longer break between the final two testing cycles because of the school's winter holiday. Data collected included (a) video (including audio) of group work and group presentations, (b) student work (i.e. group product, individual homework assignment), (c) interviews with teachers and students, and (d) field notes. Data collection procedures were directed at answering the primary research question: What factors influence student knowledge (i.e. new ideas, strategies, tool usages) diffusing within the classroom setting? Data collected in response to the primary research question led to a sub-research question: What impact does the designed teacher principles have on encouraging the development, diffusion, and appropriation of students' mathematical knowledge in the classroom?

Data collection occurred in one observational phase, two iterative testing cycles, and one validation phase. The observational phase served as an opportunity to gather general information about the classroom environment, student interactions, teacher-student interactions, and communication among students during problem solving sessions. Drafts 1 and 2 of the principles were tested across multiple MEAs in iterative cycle 1 and 2, respectively. The validation phase primarily served as a channel to validate the usability of the final set of teacher principles. During each of the four testing iterations, the principles were revised and tested again until a satisfactory set was constructed. The design team of three consisted of the lead researcher and the two participating middle school mathematics teachers. Video data were transcribed or summarized after each implementation. Data were coded and categorized according to the pattern coding method (Silverman, 2011) to specifically trace the development and diffusion of ideas among students. Phenomena from the video transcripts were coded and categorized into four situations: (1) idea shaped by group, (2) one person forces idea or practice on group, (3) misunderstanding because one member fails to understand what everyone else knows, or (4) idea based on introduction or use of classroom resources. After recording general observations from the pre-iteration 1 or observational phase, the researcher compiled a list of themes common in all of the observational phase data. Each theme was supported by specific instances or evidence from one of the data sources. This conjecture of factors that influenced the development, diffusion, and appropriation of mathematical knowledge was tested in iteration 1 and resulted in Draft 1 of the designed teacher principles. The classroom environment was modified to test Draft 1 principles in Iteration 2. The designers revised Draft 1 of the principles in testing iteration 2 based on the evidence from the themes collected from the data. Draft 2 of the principles resulted from the testing of Draft 1 of the principles. Slight modifications were made to Draft 2 of the principles after testing them in the final testing cycle, the validation stage. The final set of principles is the result of these minor revisions.

4. Findings

Observations in the pre-iteration 1 phase were purposely very broad guidelines for fostering the development, diffusion, and appropriation of knowledge. Key themes included a need for sharing ideas, strategies, and designs with students in a communal classroom environment with opportunities to test and revise students' thinking. Since the principles designed from pre-iteration 1 were mainly theoretical conjectures and only minor modifications were made to the principles during the validation stage, a detailed discussion of those iterations is omitted.

4.1 Testing Iteration 1 – Draft 1 of Principles

The preliminary teacher principles served as a framework for iteration 1 testing. Observations in iteration 1 were framed around student interactions, community development, and students' ability to reflect on the MEA task. In focusing on these specifics, a number of other factors emerged (e.g. student's use and nonuse of classroom tools, common practices associated with sharing ideas, …) as relevant observations during the testing phase of iteration 1. After comparing and triangulating data in iteration 1, a resulting list of six common themes across all data emerged. The following themes, with supporting documentation, are referred to as Draft 1 of the teacher principles.

1. Students see their group work as "their group's" work and are not willing to share any ideas or strategies with non-group members.
2. After multiple ideas are presented, students begin to work as a community by focusing on a common problem and a common solution.
3. In order for group members to share and exchange knowledge, it is necessary to consider a number of approaches, reflect on them, and attempt to revise them.
4. Getting students to respond to other students' work is a big step for students. Many students find it difficult to do so.
5. It is necessary for groups to assign a purpose for the tools or artifacts to encourage the use of tools or artifacts.
6. Students' focus is on completing the task instead of advancing knowledge. Students focus on the end-in-view instead of exploring steps to take to get to the end-in-view.

Draft 1 of the implementation principles was the beginning to examining and providing suggestions for the development, diffusion, and appropriation of mathematical knowledge among students. In testing Draft 1 of the principles, the designers' observations were focused on deficient events, communications, or interactions during the implementation of the Bigfoot MEA. This helped to better frame observations for testing the principles without first modifying the classroom environment. Evidence supporting recurring themes from analysis of iteration 1 data varies.

However, in each studied group, designers noted that students were initially not willing to share or exchange ideas or strategies with students outside of their working group. This naturally hindered the opportunity of diffusion of ideas between groups and hence was an indication of a needed change in the next testing iteration. In contrast to the next implementation, this implementation provided fewer opportunities for the public display of ideas in action. Not only were there few public trials of students working on and testing their ideas that could be observed, there were mainly only in-group instances of diffusion. After analyzing data from iteration 1, the designers opted to revise the principles such that students interacted with more than their group members and as a result the circulation of ideas was more widespread. This is reflected in the results of the Group Interaction Principle presented below.

4.2 Testing Iteration 2 – Draft 2 of Principles

The effectiveness and usefulness of Draft 2 of the principles were tested in iteration 2 through modifying the classroom environment. As in iteration 1, ideas, materials, concepts, tools, and tool-related practices were shared and appropriated between students in iteration 2. The design team purposely selected the Bigfoot MEA for the second testing iteration because of the need to encourage students to interact with members outside of their working group, as data from iteration 1 suggested. A public measuring chart was placed on a wall in the rear of the classroom. Students from all class periods measured other students, faculty, visitors, and parents and recorded the corresponding names and heights on the wall. This data was available to all students. Different groups of students interacted while congregating at the measuring chart to record data. They discussed possible relationships between height and footprint length based on comparing public data from the measuring chart, challenged one another's ideas, made estimates of height before measuring and recording, and shared problems and challenges they encountered. Individual group members often shared discussions from the measuring chart congregation area with their group members. Groups used ideas formed and discussed at the congregation area to build their solution. Sharing and building on other's ideas became important ways for the teachers to help students draw benefit from the community, rather than emphasizing a negative side of learning from others. Data from the Bigfoot MEA also revealed evidence of the diffusion of tool-related practices. One group of students examined another group's computer generated graph that showed the relationship between height and shoe size. Although the examining group was aware of how to represent numerical data in tabular representation using either an advanced calculator or computer software, they did not translate this knowledge in organizing the representations of their collected data. After seeing the usefulness of a computer generated graph, the groups engaged in a detailed discussion about the benefits of translating tabular data to a computer generated graph. Students discussed what the graph actually represented from each of their perspectives. This group successfully integrated graphing into their solution process. The physical distance between the groups, their beliefs in organizing and recognizing patterns, and their need to interpret and present their data in a convincing fashion facilitated the diffusion of a "mathematical tool".

Data analysis also revealed that a number of details were shared and appropriated between students and groups, such as tools, tool-related practices, materials, concepts, and ideas. Emily (teacher 1) revealed in an interview that her students often "cling to their own ideas", although they may be introduced to other ideas. On the other hand, once multiple ideas were presented, students negotiated strategies to advance the knowledge of the group. However, not all interactions were positive experiences for students. Collected data revealed instances of negative influences or misconceptions spreading rapidly through and between groups of students. There were two specific examples of such in the implementation of the Bigfoot activity. After groups of students incorrectly concluded that the length of one's shoe is equivalent to one's shoe size (e.g. if Jasmine wear a size 9 shoe, then her foot is 9 inches long), more than half of the groups were basing their primitive solution on this idea. The notion spread as members traveled from group to group measuring one another's shoe or merely asking for a shoe size and declaring that they had obtained the length of one's footprint, when in fact they meant they had obtained the student's shoe size. Although this fallacy slowed the final product process of many of the groups, students did eventually use logic or one another to reason that they were headed in a wrong direction. Similarly, within working groups, students arrived at conclusions that the difference in one's height and shoe length is the same for everyone. For example, many students, even as their final product, argued that one's height can be found by adding three to the shoe length since the difference in most people's shoe length and height is 3 units.

4.3 Final Set of Teacher Principles

This study provides a different framework for looking at classroom learning. According to the results of the study, it is not reasonable to claim that the final model developed by students is only attributable to the individual student. Instead, it is also a result of the diffusion of ideas throughout the classroom. The designers integrated results obtained from testing the previous set of principles with other forms of data, such as student work and teacher interviews, to obtain the final set of principles listed below. Overall, the study confirms that ideas and other knowledge components do spread to other students in classroom settings and points to five factors that influence the development, diffusion, and appropriation of knowledge. They are the (i) interaction within and between groups of students, (ii) ease of understanding and accessing resources, (iii) effort of sharing a practice, process, and product as a group, (iv) ability to compare and critique one's own solution and other's solutions, and (v) ability to redirect misconceptions.

5. Discussion of Final Set of Principles

5.1 Principle 1: Group Interaction Principle

One of the more frequent opportunities for students in the studied classrooms to learn from and share their knowledge with other students emerged in the context of their communication and mobility. When students were exposed to ideas that 'circulated' the classroom, they engaged in transactions where ideas were continuously negotiated, defended, discarded, and adapted. More importantly, designers noted that interactions *between* groups were different from those *within* groups. Advancing the knowledge of the community and individuals through the diffusion of knowledge was significantly influenced by students' communications and interactions with both members of their working group and members outside of their group. During intra-group collaborations, students focused on developing and structuring their designs and models. The inter-group environment led students to further discussions of the quality of their models, functions of their designs, and justifications for their reasoning. Both types of interactions elicited and fostered the development, sharing, and appropriation of knowledge through opportunities of peer dialogue, submission of contrasting ideas, and active seeking of information outside the group. When testing this principle, the designers instituted a classroom culture where students were required to interact with members outside of their group and ideas became seeds for the development of others. This notion of diffusion and appropriation of ideas in classrooms does not suggest imitation, but rather a process of interpretation, adaptation, and evolution. Although it may seem that because students had access to multiple ideas, strategies, and conceptual approaches, they would copy one another's solutions, they did not. As used in this study, the notion of diffusion and appropriation of ideas in classrooms does not imply copying. Instead, it is indicative of a process of interpretation, adaptation, and advancement. No instances of students copying one another's models or ideas are represented in the data.

5.2 Principle 2: Accessibility, Transferability, and Meaningfulness of Resources Principle

In classrooms, like other communities, it is valuable for students to utilize available resources or tools (e.g. graphing calculators, computing devices, student-created artifacts …) in the environment. Resources are considered to be any device or design that aids one in the solving of some problem and can include mathematical tools, facts, artifacts, rules, and heuristics. Data revealed that the development, diffusion, and appropriation of knowledge is facilitated when a resource or resource-related practice is transferable to different applications, easily accessible, highly desirable from a student perspective, well promoted by the teacher or other students, and a good fit into a system of meaningful practices. The use of surrounding resources enabled students to work on more complex problems than they would otherwise be able to do. They assisted students in thinking about and analyzing complex data in ways that reveal patterns, trends, and direction. In order to promote this usage, it is necessary for these resources to be appreciated or seen as being useful by the students. Otherwise, as revealed in the data, students tend to either not recognize the presence of the resources or view them as not being helpful in their problem solving process.

5.3Principle 3: Shared Practice, Process, and Product Principle

Knowledge creation and sharing is supported through building a community where students work toward a common goal, negotiate meanings and goals, and share an understanding of the community's ideas, strategies, designs, tools, etc. It was when students focused on communal problems of understanding that the knowledge of the group was advanced.

The group served as breeding grounds for the development and diffusion of knowledge because they provided a forum where individuals shared a variety of perspectives about a common topic, new or divergent points of view emerged and sparked innovations, and students felt comfortable sharing challenges and testing preliminary ideas.

5.4 Principle 4: Metacognitive Approach Principle

This principle reflects the metacognitive approach of advancing individual and classroom knowledge through reflecting on, testing, and, if necessary, revising one's own solution. The process of attempting to make advances on what one knows and understands leads to a deeper understanding of the content because in thinking about others' solutions, students are better able to understand their own. Also, by placing student work in the context of a larger mission, such as comparing and possibly modifying solutions, student work has a noted value that extends beyond assessment. The metacognitive approach principle emphasizes the need of not only developing and diffusing knowledge but also appropriating it through comparing alternate ideas and selecting those that are most and least useful. In this reflective process, students have ample time to reconceptualize their models based on the appropriation of ideas.

5.5 Principle 5: Negative Influence Principle

There are often both negative and positive influences on the development and diffusion of ideas in classrooms. While teachers generally welcome positive influences, negative influences tend to create an intractable problem. They spread rapidly, decrease the rate at which progress occurs, and plant seeds of faulty ideas. Therefore, to make progress, groups need to sense deficiencies in their ways of thinking. Helping students identify strengths and weaknesses in their thinking, ideas, and work can lead to advancing students' thinking.

6. Implications and Conclusion

The product of this study is a set of designed principles for teachers to consider in encouraging the development, diffusion, and appropriation of knowledge while students work collaboratively on thought revealing mathematics activities. The five principles that resulted from the analysis of data in this study are pedagogical conditions rather than requirements. They are not sufficient, in and of themselves, to design environments conducive to the development, diffusion, and appropriation of ideas. Teachers are expected to alter the principles to fit both their and the students' individual needs. Exploration of issues presented in this study is relevant to not only middle school mathematics teachers, but also researchers, curriculum developers, students, and others in the education community. A set of useful guidelines for teachers to use in encouraging the spreading of ideas among students throughout the classroom is one outcome of these explorations. This research confirms that classroom structures should facilitate the communication, justification, and elaboration of ideas in encouraging the spreading process. Also, classroom structures or norms of interactions should facilitate the interaction and collaboration between groups and de-emphasize competitiveness between groups and the notion of cheating or copying when interacting with other students. A higher rate of student academic success is a possible result of identifying effective ways of spreading understanding in the classroom. Further research on studying learning in the making rather than only the products of learning will help the education community better understand the diffusion process. Although there is a significant amount of research on within group interactions, there is little research on phenomena occurring between groups of learners in classrooms. Instead of considering interactions between groups as noise, research investigating the process in which students in groups gain access to alternative ideas through interacting with others is needed.

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