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Conceptual and Procedural Knowledge of Rational Numbers for Riyadh Elementary School Teachers

Dr. Khaled Helmi Khashan[1]

Abstract

The present study aims to investigate the conceptual and procedural knowledge of Riyadh primary school mathematics teachers' regarding rational numbers. The sample of the study consisted of 57 mathematics teachers in Riyadh primary schools during the second semester of the academic year 2012/2013. More specifically, the sample consisted of 27 novice teachers and (30) experienced teachers. To achieve the objectives of the study, conceptual and procedural knowledge regarding rational numbers were tested by an instrument with well-established reliability and validity. The findings of the present study indicated that mathematics teachers in Riyadh primary schools have average conceptual and procedural knowledge regarding rational numbers. In addition, there was statistically significant difference at ($\alpha = \leq 0.05$) between the mean scores of teachers' conceptual knowledge and the mean score of their procedural knowledge. The difference was in favor of the procedural knowledge. Moreover, the study found that there was a statistically significant difference at ($\alpha \leq 0.05$) between the mean scores of novice teachers and experienced teachers. This difference was in favor of experienced teachers. It also revealed that there was a non-statistically significant weak positive correlation between teachers' conceptual knowledge and their procedural knowledge regarding rational numbers.

Keywords: Rational Numbers, Conceptual Knowledge, Procedural Knowledge

Theoretical Framework and Literatures

Mathematical knowledge occupies a prominent place in human knowledge, and is a prerequisite in all aspects of life. It has become the fuel that drives the movement of society to move forward without obstacles because of its wide applications and neccessity in an individual's life.

The effective learning and teaching processes of mathematics include not only conveying all the facts from teachers to students, but also providing opportunities for teachers to improve their conceptual and procedural knowledge about each subject they teach (Pa & Aziz, 1992). U.S. National Council of Teachers of Mathematics (NCTM, 2000) confirmed that teachers need to encourage their students to develop their knowledge of mathematics through inquiry, exploration, examination of hypotheses, approximation, problem-solving process, research, and discussion of ideas. Therefore, teachers must instill the conceptual knowledge in the minds of their students so that they would have adequate competence in order to solve all types of problems and tasks

[1] King Saud University

Conceptual knowledge is in general an abstract knowledge addressing the essence of mathematical principles and relations among them, while procedural knowledge consists of symbols, conditions, and processes that can be applied to complete a given mathematical task (Hiebert & Lefevre, 1986). They also confirmed that procedural knowledge is meaningful only if it is linked to a conceptual base. Faulkenberry (2003) suggests that conceptual knowledge is rich with relations, and refers to the basic mathematics constucts and relations between the ideas that illustrate mathematical procedures, and gives it a meaning. On the other hand, procedural knowledge addresses the mastery of mathematical skills, acquaintance of the procedures to determine the mathematical components, algorithms, and definitions. Procedural knowledge is viewed by (McGehee, 1990) as the ability to explain or justify the way one resolves a given problem without knowing the reason behind applying a certain theory, process, or law during problem-solving process. Mahir (2009) indicates that conceptual knowledge is correlated to other knowledge sets, this knowledge allows the individual to distinguish between these correlations, which are as important as the sets themselves, while procedural knowledge is the official language of mathematics, rules, and procedures used in mathematical problem-solving processes.

Teaching procedural knowledge means teaching terminology, symbols, and different skills without focusing first on building a deep understanding relating to the terms, symbols, and skills taught to the associated concepts (Skemp, 1987). On the other hand, teaching conceptual knowledge begins by giving students a problem requiring flexibility of thinking and making relations to what they previosuly learned as they progress in steps of problem solving, allowing them to expand their prior knowledge and apply it in new situations (NCTM, 2000). Byrnes & Wasik (1991) suggest that conceptual knowledge means that a student must view mathematics as a meaningful topic, and that the difference between learning for conceptual knowledge and traditional learning lies in that the latter ends each chapter with applied problems on principles, laws, and algorithms contained in that chapter, while the former usually introduces applications at the beginning of the chapter and then relations and principles are concluded. Although both ways of learning agree that conceptual knowledge and procedural knowledge are basic and important components, however, the basic idea for both lies in the way and order by which each of them is taught. Learning conceptual knowledge first leads to the acquisition of procedural knowledge, but not vice versa.

Mousad (2006) indicates that conceptual knowledge is reflected through individual's ability to produce what could be considered examples and what couldn't be considered examples of concepts; use of shapes and graphics to express concepts; use of mathematical, manual, technological, and intellectual processing; besides modeling concepts and translating them into denotations and ideas explaining the mathematical system through using codes, phrases, and relationships for conceptual communication. Many researchers indicate that both conceptual knowledge and procedural knowledge are important components in understanding mathematics (Desimone et al., 2005; Hiebert et al., 2005). Therefore, both types of knowledge must be presented when teachers convey a mathematical content.

Despite a long history of research on the relationship between conceptual knowledge and procedural knowledge, the findings are not categorically conclusive, as divergent views about this relationship are highlighted. They did not agree on a specific position in terms of its nature. Each of these views is supported by some empirical evidence (Hallett et al., 2010). For example, Anderson's learning model points out that acquisition of procedural knowledge is based on the existence of conceptual knowledge and knowledge gained by frequent use of procedures (Anderson, 1995), and hence conceptual knowledge is transformed into procedural knowledge (Byrnes and Wasik, 1991). As for Piaget, the situation is different.
He suggests that after the student acquires mastery of procedural knowledge, a reflection process begins, resulting in new conceptual knowledge (Byrnes & Wasik, 1991), meaning that he views both conceptual knowledge and procedural knowledge as an integral part of the individual's cognitive schema, that both types of knowledge are not separate, and that the procedural effectiveness is a prerequisite for conceptual knowledge (Baker & Czarnocha, 2002).

This viewpoint was supported by Baker and others (Baker et al., 2004), as they suggest that the development of conceptual knowledge in traditional approaches is regarded as a product for computational effectiveness of the procedures associated. Aspinwall and Miller (1997) also concurred on this view, as their study showed that students consider the development of their procedural knowledge basicaly a result of their studies of the course of differentiation, so they finish this course with little conceptual understanding.

The dynamic action view is considerd as an example of models that are based on procedural knowledge first. By virtue of this model, learning takes place through the application of procedural knowledge based on the pre-existing conceptual knowledge. In this case, the higher efficiency in procedural knowledge helps to further expand the conceptual knowledge (Haapasalo & Kadijevich, 2000).

A third view on this issue is adopted by (Gray & Tall, 1994). They suggest that the successful thinker in mathematics uses a mental complex composed of a combination of processes and concepts that are called (procepts), i.e. a merge between procedural processes and conceptual knowledge, or what is called a conceptual procedure. A successful thinker has flexibility of thinking that allows him to move between procedural way to perform a mathematical task and its concept that has been mentally processed as part of a wider plan. Researchers pointed to four basic stages to gain procedural knowledge that is based on conceptual knowledge, namely:

- Pre-procedures Stage: Where a learner has unorganized procedural knowledge.
- Procedure Stage: Where a learner has one routine modality to solve problems.
- Process Stage: Where a learner has many, flexible, and diverse modalities to solve one mathematical problem.
- Conceptual Procedure Stage (Percept): Where learner's procedural thinking is based on a sound conceptual basis, i.e. where the learner is capable of coding mathematical thinking, underlying association, representation of concepts, generalizations, and mathematical algorithms.

This point of view is supported by (Rittle-Johnson & Alibali, 1999), as they believe that the conceptual knowledge and procedural knowledge develop across nested and interrelated processes. In other words, the development in one knowledge reflects positively on the other, and so they are developing simultaneously. Rittle-Johnson and Koedinger (2005) suggest that simultaneous sequence between conceptual knowledge and procedural knowledge, as an educational strategy, leads to improved and developed conceptual and procedural knowledge more than simultaneous sequence between both of them based on precedence of conceptual knowledge to procedural knowledge. The effect of simultaneous sequence on the procedural knowledge is better than the sequence based on precedence of conceptual knowledge to procedural knowledge.

Star (2002) adopts this view too, as he suggests that procedural processes learned through the routine way is more prone to be easily forgotten, and therefore it is a wrong way of teaching. Learning procedural processes must be linked to learning conceptual knowledge on which it is based in order to understand them.

Baker and Czarnocha (2002) concluded that conceptual knowledge is independent of the individual's ability to apply procedural knowledge. This view is echoed by (Vygotsky, 1986) who suggests that the development of conceptual knowledge proceeds through the reflection of pre-existing conceptual knowledge independently of reflection resulting from the repetition of procedures and algorithms.

Students learn math through their gained experience and exercises of solving mathematical problems in the classroom. Thus, teachers should first master the mathematical concepts they teach, and move away from rote methods in their teaching of these concepts.

A number of studies have shown that most teachers do not have a good understanding of the mathematical content which they teach (see Stump, 1996; and Mahir, 2009). Al Nazeer (2004) indicates that there is a significant weakness among mathematics teachers in Saudi Arabia in the field of development of mathematical concepts. Rashid and Khasan (2009) supported the opinion that the main obstacles to mathematics teaching from the viewpoint of supervisors is the rote teaching approach used by teachers. Some studies have shown that students are mostly dealing with mathematical content as a procedural knowledge without focusing on the conceptual knowledge of it (Khasawnah and Barakat, 2007).

A study carried out by (Miqdadi et al., 2013) concluded that undergraduate teachers at the University of Yarmouk did not achieve the level of proficiency set by the jury on the test on fractures, where the mean performance falls around pass points 50%. Findings of that study also indicate a statistically significant variation between the mean performance of the sample on the test of conceptual knowledge and the mean of their performance on the test of procedural knowledge for the favor of procedural knowledge. Findings of a study carried out by Al Souly (2013) indicate that the secondary school teachers in Saudi Arabia have an average degree of conceptual knowledge and are unable to use the simple facts and relationships when they are given in new contexts.

Conceptual knowledge and procedural knowledge were investigated by (Mahir (2009) with a group of students who have succeeded in calculus course after studying for a year at the University of Anatolia, Turkey. Five problems were given to the study subjects then their answers were analyzed accurately and in detail. Findings have revealed that students do not have a satisfactory grasp of conceptual knowledge regarding integration. Additionally, it was also shown that students who have good conceptual knowledge had a good performance in procedural knowledge too.

Zakaria and Zaini (2009) conducted a study to investigate Malaysian pre-service teachers's conceptual and procedural knowledge regarding relational numbers. The study sample consisted of one hundred and five teachers from three teacher-training colleges. The results indicated that the level of knowledge of conceptual and procedural knowledge among pre-service teachers was above average. Furthermore, teachers have shown efficiency in the representation of fractions as a part of a set, a region, and a ratio. They have also used their conceptual knowledge to draw the one set when they are given a fraction representing a part of it, and in solving life problems including fractions.

A study was conducted by (Engelbrecht et al., 2005) to examine the relationship between conceptual knowledge and procedural knowledge as well as the relationship between students' confidence level when dealing with conceptual problems and confidence level when dealing with procedural problems. The study sample consisted of 235 students who had completed the Introduction to Calculus course. The study utilized a testing instrument of a test including ten paragraphs, half of them were devoted to measure conceptual knowledge while the other half were devoted to measure procedural knowledge.

The study found out that students' performance on conceptual knowledge was better than that on procedural knowledge. It was also shown that students' confidence in their ability to answer conceptual problems was greater than their confidence in the ability to answer procedural problems.

In a study on (170) teachers in Kutabora, Malaysia, Ibrahim (2003) found out that 78.6% of inexperienced teachers have a low level of conceptual knowledge, compared with 60.7% of experienced teachers having a high level of conceptual knowledge. It also found that inexperienced teachers possess adequate procedural knowledge in mathematics, which indicated that a good math teacher is the one who provides organized and sequenced steps for the computations given. Faulkenberry (2003) found that explanations provided by inexperienced teachers was primarily based on procedural knowledge, while explanations provided by experienced teachers was based on a mixture of both conceptual and procedural knowledge. Furthermore, experienced teachers also provided a variety of ways to help students understand the concepts, and offered numerous activities to ensure that students are involved in the learning process.

A study was conducted by Bryan (2002) on nine of the pre-service teachers getting practical training in a high school to investigate whether they have acquired a good knowledge of teaching mathematics as they graduated from the university. The results showed that 37% of those teachers failed to use conceptual knowledge to clarify or justify their answers, while only 26% of them succeeded in the application of conceptual knowledge on their answers. In interviews conducted with those teachers, eight of them failed to use or display their conceptual knowledge of trigonometry and they acknowledged that everything they had learned in the school was spoon-fed. Furthermore, they were confused in interpreting the following identity: $\sin^2 x + \cos^2 x = 1$, demonstrating their weak conceptual knowledge regarding the topics in question.

Another study was conducted by (Star, 2002) on three pre-service teachers at a community college in New York who were pursuing an Algebra course. The teachers were asked to solve mathematical equations requiring conceptual and procedural knowledge. Findings revealed that procedural knowledge in solving mathematical equations increases when conceptual knowledge is at high levels.

In addition, in spite of their wrong conceptual knowledge, teachers were able to apply a variety of mathematical concepts when solving the equations. The findings also indicate their ability to use procedural knowledge which is derived from conceptual knowledge when solving algebraic equations.

In her study of division processes, including fractures, Tirosh (2000) found that pre-service teachers managed to solve calculations including fractures but failed to offer clarifications and explanations of the procedures and algorithms used in the completion of a mathmetical problem, such as the algorithm "invert and multiply" used in dividing fractions. Subjects' procedural knowledge considerably surpassed their conceptual knowledge, as they believe that teaching algorithms is enough for students to be able to grasp the mathmetical concept. Due to their lack of conceptual knowledge, those teachers could not build activities related to the concepts of fractions.

A study was run by Stump (1996) to compare between mathematical knowledge of pre-service teachers and in-service teachers concerning calculations of inclination. Tests showed that the study subjects had difficulties not only to distinguish between linear equation and other equations, but also in answering the questions related to the themes of change rate and trigonometric representations of inclination. However, the responses of in-service teachers were better than that of their pre-service counterparts, as they demonstrated better conceptual knowledge and provided more explanations and clarifications when presenting a given topic.

Furthermore, McGehee (1990) reported that most teachers have mastered procedural knowledge more clearly than their mastery of conceptual knowledge. While their performance was good in the subjects requiring the application of procedural knowledge, they faced difficulties in problems requiring conceptual knowledge.

A review of the literature clearly shows that most teachers do not have a good understanding of the mathemetical content in general, rational numbers in particular. This poor performance of math teachers in rational numbers can cause serious problems as they will eventually teach mathematics in school and convey their knowledge to their students.

This raises the question why many students and teachers have difficulties in understanding rational numbers? Vergnaud (1983) attributes that to the fact that skills associated with rational numbers rely heavily on other basic elements of knowledge. On the other hand, the topic of rational numbers is multi-faceted (Behr et al, 1983). It includes several concepts, such as the concept of a whole set, the concept of a part of a set, a region and a ratio, fractions, and decimals.

Liu (1993) reported another reason for the difficulties faced by teachers regarding rational numbers, namely their lack of ability to solve non-routine problems related to a given content.

Study Problem

Students are learning math through experiences and problems provided by their teachers in classrooms. In order to ensure effective learning of mathematics, teachers should master the mathematical content that they teach. Despite the lack of studies on the relationship between teachers' perfection of mathematical knowledge and their students' attainments, most of the few studies conducted indicated a positive relationship between the two variables (Hill et al., 2005). Hill and colleagues (Hill et al., 2005) brought attention to the importance of striving to improve students' achievements by enhancing mathematical knowledge among teachers. Furhtermore, Hammond (Hammond, 2000) goes beyond that to indicate that teachers' mathematical knowledge nowadays adays have become the basic foundation to raise the level of student achievement.

Despite the importance of mathematics teachers' acquisition of procedural knowledge, they should acquire deep conceptual knowledge in order to be able to teach their students effectively, as a number of studies have highlighted that most of mathematics teachers don't have good understanding of conceptual knowledge and procedural knowledge (Stump, 1996; Mahir, 2009).

Educational literature indicates that teachers are often unable to provide conceptual explanations of algorithms and procedures they use to complete mathematical problem-solving processes (Tirosh, 2000). Several misconceptions among mathematics teachers in primary and secondary schools were monitored in giving explanations and clarifications of mathematical procedures they use when introducing some basic mathematical knowledge, such as division, verification, and associations.

Therefore, mathematics teachers must have a profound understanding of the topics they teach (Schoenfeld, 2002; Shulman, 1986) because superficial understanding of the basic concepts isn't enough. Therfore, the purpose of this study is to explore math teachers' conceptual and procedural knowledge of rational numbers at the level of the Primary cycle in Riyadh.

Study Questions

The study seeks to answer the following four questions:

1. How far mathematics teachers in Primary schools in Riyadh master conceptual and procedural knowledge in the subject of the rational numbers?
2. Is there a statistically significant difference at ($\alpha \leq 0.05$) between the mean scores for mathematics teachers in Primary schools in Riyadh in conceptual knowledge and the mean of their scores in procedural knowledge in a test on conceptual and procedural knowledge?
3. Is there a statistically significant difference at ($\alpha \leq 0.05$) between the mean of the scores of novice mathematics teachers in Primary schoolS in Riyadh and the mean of scores of experienced counterparts in a test on conceptual and procedural knowledge?
4. Is there a statistically significant correlation at ($\alpha \leq 0.05$) between math teachers' conceptual knowledge and their procedural knowledge regarding rational numbers in Primary schools in Riyadh?

Importance of the Study

Increasing student's achievement in mathematics is one of the goals of the Ministry of Education in Saudi Arabia. A large number of policymakers believe that achieving this goal depends largely on the quality of teaching provided. Researchers and policymakers tend to agree that the quality of teaching relies heavily on teachers' understanding of the subject he teaches (Ball, 1990; Ma, 1999; Kilpatric et al., 2001).

In addition, students' understanding regarding rational numbers is the basis of their development in the field of mathematics, as this topic is regarded as one of the most difficult topics facing both the teacher and students (Zakaria, 2002; Simoneux et al., 1997).

Educational research has demonstrated that students' responses to mathematical problems are determined by their intuitive concepts which are often not compatible with mathematical definitions and theories, leading them to acquire misconceptions about the rational numbers. This research has also shown that it is almost impossible to correct misconceptions (Simoneaux et al., 1997; Tirosh & Graeber, 1990). Therefore, it is possible to assume that most of these teachers learned these misconceptions in their early stages of learning. The importance of not to convey these misconceptions to students is further emphasized.

Here lies the importance of the present research that will reveal math teachers' conceptual and procedural knowledge regarding rational numbers at the level of primary school in Riyadh. Study findings may be useful for those in charge of teachers' preparation and training programs so that they can use teaching methods focusing on conceptual knowledge, developing programs designed to increase the teaching capacity of teachers of this stage, and train them to focus on conceptual knowledge particularly while teaching rational numbers.

Study Terms and Their Procedural Definitions

Conceptual Knowledge

The outcome of individual's knowledge of underlying relations and associations between ideas that explain and give meaning to mathematical procedures. Conceptual knowledge was measured for each of the sample subjects by the score he obtained in conceptual knowledge problems contained in conceptual and procedural knowledge test prepared by the researcher for this specific purpose.

Procedural Knowledge

The outcome of individual's ability to solve the problem by processing mathematical skills, such as procedures, rules, laws, algorithms, and codes used in mathematics. Procedural knowledge was measured for each member of the sample by the score he/she obtained in procedural knowledge problems contained in conceptual and procedural knowledge test prepared by the researcher for this specific purpose.

Novice Mathematics Teachers At Primary Cycle

They are those teachers who have been teaching mathematics at the primary level since two years or less.

Experienced Mathematics Teachers at Primary Cycle

They are those teachers who have been teaching mathematics at the primary level for more than five years.

Study Limitations

Study findings can be generalized in light of the following limitations:

- Study sample was limited to a group of mathematics teachers of primary cycle in Riyadh, Saudi Arabia during the second semester of the academic year 1433/1434 AH (corresponding to 2012/2013) .
- Study sample was limited to novice teachers as well as experienced teachers.
- Study instrument is an instrument that has been developed specifically for the purposes of the study. Therefore, the interpretation of the findings depends largely on its reliability and consistency, knowing that its validity and reliability have been duly and appropriately verified and assured.

Study Methods, Techniques, and Procedures

Methods and procedures included a description of the study sample and what has been done by the researcher to prepare the study instrument, in addition to the procedures he had made to measure how far mathematics teachers at the primary level in Riyadh master conceptual and procedural knowledge in terms of rational numbers.

First: Population and Sample

Study population encompasses all mathematics teachers in primary cycle in Riyadh, Saudi Arabia in the second semester of the academic year 2012/2013. Study sample consists of (57) math teachers at the primary cycle in Riyadh in the second semester of the academic year 2012/2013, of whom (27) are novice teachers, while the rest (30) are experienced teachers.

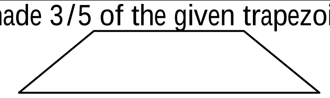
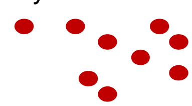
Second: The Instrument of the Study

To achieve the objective of this study, a test on conceptual and procedural knowledge regarding rational numbers was designed based on conceptual and procedural knowledge test regarding rational numbers designed by (Faulkenberry, 2003). The researcher translated this test into Arabic and added some mathematics problems to suit the syllabus of primary schools in Saudi Arabia. Thus, the number of questions increased to 34 of the construction type requiring solutions and clarifications of the calculations made to reach the answer. Half of the questions were designed to measure conceptual knowledge among the study sample while the other half were designed to measure their procedural knowledge. Given the lack of clear-cut boundaries between conceptual knowledge and procedural knowledge in mathematics, the researcher took into account the selection of each question that it is designed to measure the respective knowledge to be measured. The test covers seven sub-topics, namely: the concept of a part of a set, order, fraction equivalence, the concept of a set, processes on fractions, life applications of rational numbers, and questions specific to rational numbers.

Test Content Reliability

To ascertain reliability of the content of the test in measuring both conceptual and procedural knowledge in the sample, it was introduced, in its initial form, to a group of experienced arbitrators and specialists in mathematics, curricula, and teaching methodology. They were asked to assign a percentage for each question to reflect its relative weighting ratio of measuring the respective knowledge for which it is designed, and modify any of the questions as necessary to be more accurate in measuring the respective knowledge for which they were designed. Questions with average estimate of less than 70% were deleted and then proposed amendments were made. The final form of the test consists of 30 questions, a half of which measures the conceptual knowledge while the other half measures the procedural knowledge. Table (1) shows an example of each sub-topic of rational numbers and the type of knowledge that is measured.

Table 1: An Illustrative Example of Each of the Sub-Topics of Rational Numbers and the Type of Knowledge that is Measured

Para	Sub-Topic	Type of Knolwedge	Example
1	Concept of part of a set	Conceptual Knowledge	Shade 3/5 of the given trapezoid 
2	Order	Procedural Knowledge	$\frac{2}{3}$ $\frac{3}{4}$ 0.9 $\frac{11}{12}$ $\frac{16}{31}$ $\frac{9}{19}$
3	Fractions Equivalence	Procedural Knowledge	Complete the blank spaces $\frac{3}{8} = \frac{\_\_}{12}$ $\frac{8}{14} = \frac{\_\_}{21}$ $\frac{8}{15} = \frac{\_}{5}$
4	Concept of Unit	Procedural Knowledge	The following circles represent $\frac{3}{7}$ of a given unit. How many circles in a unit? 
5	Processes on Fractions	Procedural Knowledge	Find the outcome of the following: 1. $1\frac{3}{4} \div \frac{1}{2}$ 2. $1\frac{3}{4} \div \frac{1}{2}$ 3. $1\frac{3}{4} - \frac{1}{2}$ 4. $1\frac{3}{4} + \frac{1}{2}$
6	Life applications on fractions	Conceptual Knowledge	A designer is in need of $3\frac{3}{4}$ rolls of wall paper to decorate a room. How many rooms can he decorate using 13 rolls of the same wall paper?
7	Specific problems	Conceptual Knowledge	What happens to this fraction $\frac{9}{8}$ if the nominator is increased by three folds and the numerator is divided by 4?

Test Consistency

To measure the consistency of conceptual and procedural knowledge test, it has been applied to a reconnaissance sample of the study population outside of this study sample, consisting of 21 teachers. Internal consistency coefficient, represented by Cronbach's alpha coefficient, was found to be equal to 0.86, which is appropriate enough to achieve the objectives of this study.

Marking The Test

The researcher requested respondents, by means of the test instructions, to solve the questions clearly without oversight of any of the sketches or the steps. Each question was assigned a maximum score of four 4 points. Accordingly, the maximum score of each of the conceptual knowledge part and the procedural knowledge part is 60 separately, while the minimum score is 0. The maximum score of the overall test is 120 while the minimum score is 0. Then, each was transformerd to be 100. The scale described in Table 2 below is used to mark respondents' responses to conceptual knowledge and procedural knowledge.

Table 2: Marking Scheme for Subjects' Responses to Conceptual Knowledge and Procedural Knowledge Questions

Description of Response (Performance)	Quantification
Ideal Answer; the teacher provides a complete answer with a consistent, clear, and integrated explanation; containing a clear and simplified sketch through which he communicates effectively with whoever checks the answer; he demonstrates understanding of ideas and the relationships between them; then he identifies the important elements in the problem, and gives supporting and opposing examples as well as strong and supporting arguments for his answer.	4
The answer contains small mistakes, yet satisfactory; he completes the solution satisfactorily but the explanation is not clear and arguments are sometimes incomplete; the scheme may be insuitable or unclear; he understands the underlying mathematical ideas and uses them effectively.	3
The answer contains small mistakes, yet almost satisfactory; he begins solving the problem properly but he may fail to complete it; He may ignore important aspects of the problem and fail to demonstrate a full understanding of mathematical ideas and processes; he may make major mistakes; he may fail to use mathematical terms; the answer may reflect inappropriate strategy in problem-solving process.	2
He begins the solution then he fails to complete it; explanation and clarification is not understood; the sketch may be unclear; he doesn't show any understanding of the problem; he may make many computation mistakes.	1
He can't start in the solution effectively; the words contained in the solution don't reflect the problem; sketches don't represent the situation in question; he copies parts of the problem without attempting to solve it; he also fails to identify any appropriate information on the problem.	0

Third: Assessment the Extent to Which Subjects Master Conceptual and Procedural Knowledge

To assess the extent to which subjects master conceptual and procedural knowledge as a whole, and assess how far they master each of these two types of knolwedge seperately, the mean of their scores in each of the knowledge types were categorized into four levels: very low, low, average, and high. Table 3 below illustrates the criteria for this categorization (Teacher Education Division, 2000).

Table 3: Classification Criteria of Subjects' Scores

Serial No.	Overall Score in the Test as A Whole or in One of the Two Parties (%)	Mastery Degree of Conceptual knowledge or Procedural or Both Together
1	Less than 50	Very Low
2	50-64	Low
3	79-65	Average
4	80 and above	High

Fourth: Data Collection

To achieve the objectives of this study, conceptual and procedural knowledge test was applied to all subjects in the second semester of the academic year 2012/2013, and their responses were marked using the scale shown in Table 2 above.

Fifth: Statistical Processing

After subjects' answers on conceptual and procedural knowledge test were marked, the following statistical processes were used in order to answer the study questions:

1. To answer the study's first question, the mean of subjects' scores in conceptual knowledge and their mean scores in procedural knowledge questions were calculated separately, and then their mean scores in both as a whole.
2. To answer the study's second question, (t-test) was used to examine the significance of the difference between subjects' mean scores in conceptual knowledge portion and their mean scores in procedural knowledge portion of conceptual and procedural knowledge test.
3. To answer the study's third question, t-test was run to examine the significance of the difference between the mean of the scores of novice math teachers in primary school in Riyadh and the mean of the scores of their experienced counterparts in conceptual and procedural knowledge test.
4. To answer the study's fourth question, Pearson correlation coefficient was calculated between the mean of the scores of the respondents for their answers to conceptual knowledge questions and the mean of their scores for their answers to procedural knowledge questions.

Study Findings and Discussion

Firstly: Findings Related to the First Question

To answer the first question of study, the researcher calculated the means and standard deviations of teachers' scores in each of the conceptual knowledge portion and the procedural knowledge portion and in the test as a whole. Table 4 shows these means and standard deviations.

Table 4: Means and Standard Deviations of Subjects in Both Conceptual Knowledge and Procedural Knowledge and in the Overall Conceptual and Procedural knowledge Test

Serial No.	Type of knowledge	Mean (%)	Standard Deviation
1	Conceptual Knowledge in Rational Number	61.78	7.25
2	Procedural Knowledge in Rational Number	74.62	7.47
3	Overall knowledge in Rational Number	68.20	5.50

Using the cateogorization criteria of the scores shown in Table 3, it is shown from Table 4 that teachers' mean scores regarding their conceptual knowledge portion about rational numbers is 61.78%, and their standard deviation is 7.2, suggesting that mathematics teachers at the primary cycle in Riyadh have low-grade conceptual knowledge regarding rational numbers.

Table 4 also shows that teachers' mean scores regarding their procedural knowledge portion about rational numbers is 74.62 %, and their standard deviation is 7.47, indicating that math teachers at the primary cycle in Riyadh have average procedural knowledge regarding rational numbers. As shown in Table 4, the mean of subjects' scores in conceptual and procedural knowledge test regarding rational numbers is 68.20 % and their standard deviation is 5.50, suggesting that math teachers at the primary cycle in Riyadh have average conceptual and procedural knowledge regarding rational numbers. This, in turn, indicates the weakness of math teachers at the primary cycle in Riyadh in both conceptual and procedural knowledge regarding rational numbers, despite the fact that their procedural knowledge was better than their conceptual knowledge.

The finding may be attributed to the conventional methods and techniques by which the subjects were taught along their learning process, featuring the memorization of information and the automatic application of processes without focusing on mathematical concepts and conceptual structures . They did not learn rational numbers in ways that help them to understand and justify the knowledge, which prevented the formation of deep conceptual knowledge in this topic. This, again, made their procedural knowledge in this regard prone to forgetfulness. This was confirmed by the U.S. National Council of Teachers of Mathematics (NCTM, 2000) when it considered modern teaching strategies make learning math meaningful.

The results can also be due to subjects' teachers lacking sufficient understanding of the nature of mathematical knowledge and its structure. This was confirmed by Al Nazeer (2004) as he pointed to a significant weakness among mathematics teachers in Saudi Arabia regarding the development of mathematical concepts, and by Rashid and Khashan (2009) when they pointed out that the leading obstacle to mathematics education teaching from the viewpoint of supervisors is teachers' adoption of the method of memorization.

A review of literature reveals that this finding concurred with some previous studies by Miqdadi and colleagues (2013), Al Selouly (2013), (Mahir, 2009), and (Stump, 1996) that pointed to the low level of mathematics teachers in conceptual and procedural knowledge. On the other hand, it was found that this finding is inconsistent with the study by (Zakaria & Zaini, 2009) which indicated that pre-service teachers' conceptual and procedural knowledge is above average, and with the study by (Ibrahim, 2003) which indicated that inexperienced math teachers' procedural knowledge is high.

Second: Findings Related to the Second Question

To answer the second question of study, subjects' scores in conceptual and procedural knowledge test was run into the Statistical Package for the Social Sciences (SPSS), then the mean and standard deviation were calculated and were as shown in Table 5.

Table 5: Means and Standard Deviations of Subjects' Scores in Conceptual Knowledge and Procedural Knowledge

	Number of Teahcers	Mean	Standard Deviation
Conceptual Knowledge	57	61.78%	7.25
Procedural Knowledge	57	74.62%	7.47

Table 5 shows that there is a difference of 12.84 % between the mean of teachers' scores on the conceptual knowledge portion of the test 61.78 % and the mean of their scores on procedural knowledge portion 74.62 %. This difference was in favor of the procedural knowledge. In order to test the significance of this difference, t-test was run and the results were as shown in Table 6.

Table 6: **Test Results of (t) Test of Subjects' Scores in Conceptual Knowledge Portion and Procedural Knowledge Portion of Conceptual and Procedural Knowledge Test**

Group	Mean	degrees of freedom	(t) Value	Significance Level
Conceptual Knowledge	66.78%	112	9.31	0.000
Procedural Knowledge	74.62%			

Table 6 shows that the calculated value for (t) is 9.31 and that its significance level is 0.000 which is statistically significant at the level of ($\alpha = 0.05$), suggesting a statistically significant difference, at the level of significance ($\alpha = 0.05$), between the subjects' mean scores in conceptual knowledge portion and their mean scores in procedural knowledge portion in favor of the latter. This leads us to believe that math teachers at the primary cycle in Riyadh have a low level of conceptual knowledge, while they are more able to apply the procedural knowledge than conceptual knowledge, suggesting that they don't have sufficient understanding of the nature of mathematical knowledge and its structure.

This finding may be due to the fact that what students had learned in mathematics are just the rules and procedures memorized by heart and used as algorithms to solve problems, without understanding these rules and procedures or the relationships between them.

This have created stereotypes of the concept in their minds through dealing with the applications of that concept, rather than by problems focusing on the concept itself, as a result of the quick introduction of the concept to them and then moving directly to mathematical problem-solving process. In other words, they spent most of their time in schools dealing with procedures with little focus on conceptual knowledge upon which these procedures have been built.

This can also be attributed to conventional methods prevalent in our educational institutions, by which presently teachers were taught along their earlier learning cycles, clearly focusing on procedural knowledge at the expense of conceptual knowledge. This, in turn, didn't give a room for teachers to deal with conceptual problems.

Acquisition of profound conceptual knowledge of mathematics needs adopting teaching strategies that focus on helping learners to identify the relationships between mathematical ideas, understand the nature of the bonding between these relationships, and how they are built upon each other in order to produce both an integrated and coherent set; and focus on the applications of those ideas in mathematics and beyond. This was also stressed by Darey and his colleagues (Darey et al., 2012) when they pointed out that students are dealing with mathematical content mostly as procedural knowledge without focusing on conceptual knowledge, making their mathematical experience concentrated on procedural knowledge more than conceptual knowledge.

Likewise, it was confirmed by Miqdadi and colleagues (2013) as they pointed out to conventional techniques used in teaching mathematics that focus on the procedural aspect, without focusing on the conceptual structure, where teachers mostly direct their students to learn by means of focusing on computation processes without concentrating on understanding.

Furthermore, it can also be attributed to the evaluation techniques used in education and syllabus, whether it be in general education or in higher education. These techniques focus on evaluating learners' procedural knowledge more than their concern with conceptual knowledge, resulting in lack of opportunities for learners to gain conceptual knowledge as they should, and not to deepen their understanding of mathematical concepts regarding rational numbers. This was confirmed by Selouly (2013) who reported that the evaluation techniques used in public education and higher education play a prominent role in the weakness of learners' conceptual knowledge, because these strategies place learners in a narrow circle of ready-made situations of procedural processed they learned and memorized.

A review of the pedagogics literature revealed that this finding is consistent with the findings of some previous studies indicating that math teachers' ability to apply procedural knowledge is greater than their ability to apply conceptual knowledge, (see Miqdadi et al., 2013; Zakaria & Zaini, 2009; Star, 2002; Bryan, 2002; Tirosh, 2000; McGehee, 1990). This finding, however, is inconsistent with the findings of the study by (Engelbrecht et al., 2005) which indicated that students' performance on paragraphs measuring conceptual knowledge was better than their performance on paragraphs measuring procedural knowledge.

Third: Findings Related to the Third Question

To answer the third question of study, subjects' scores in conceptual and procedural knowledge test were run into the Statistical Package for the Social Sciences (SPSS), then the mean and standard deviation were calculated for the scores of novice teachers and experienced teachers in the conceptual and procedural knowledge test, and were shown in Table 7.

Table 7: Means and Standard Deviations of novice and experienced Teachers' scores in Conceptual and Procedural Knowledge Test

	Number of Teachers	Mean	Standard Deviation
Novice Teachers (with less than 2 years of experience)	27	66.17%	4.65
Experienced Teachers (with more than 5 years of experience)	30	70.03%	5.63

Table 7 shows that there is a difference between the mean scores of novice teachers and the mean scores of their experienced counterparts in conceptual and procedural knowledge test. Mean scores of novice teachers is 66.17%, while the mean scores of highly experienced teachers is 70.03%, with a difference of 3.86% between them in favor of experienced teachers. In order to test the significance of this difference, t-test was run and the results were as shown in the Table 8 below.

Table 8: Test Results of (t) Test of Subjects' Scores in Conceptual and Procedural Knowledge Test

Group	Mean	degrees of freedom	(t) Value	Significance Level
Novice Teachers Teachers	66.17%			
Experienced Teachers	70.03%	55	2.80	0.007

Table 8 shows that the calculated value for (t) is 2.80 and that its significance level is 0.007 which is a value of statistical significance at the level of significance ($\alpha = 0.05$), suggesting a statistically significant difference, at the level of significance ($\alpha = 0.05$), between the mean scores of conceptual and procedural knowledge of novice math teachers in primary cycle in Riyadh and that of their experienced counterparts. This difference is in favor of experienced math teachers.

This finding may be due to the fact that novice mathematics teachers at the primary cycle did not practice teaching long enough to develop their conceptual and procedural knowledge regarding rational numbers. This was confirmed by (Stump, 1996) who concluded that in-service math teachers' mathematical knowledge is better than their pre-service counterparts. This was also found true by the study conducted by (Faulkenberry, 2003) as its findings indicated that explanations provided by novice teachers are primarily based on procedural knowledge, while interpretations provided by experienced teachers are based on a mixture of conceptual and procedural knowledge. This finding was also echoed by the study run by (Ibrahim, 2003) which demonstrated that experienced teachers have a better command of conceptual knowledge than their inexperienced counterparts.

Fourth, Findings Related to the Study Fourth Question

To answer the fourth question of the present study, Pearson correlation coefficient between subjects' mean scores in conceptual knowledge portion and their mean scores in procedural knowledge portion was calculated and found to be 0.12 with a significance level of (0.39) which is considered a small positive correlation value of no statistical significance at the level of significance of (0.05), suggesting the absence of function correlation between math teachers' conceptual knowledge and procedural knowledge at the primary cycle in Riyadh; in the sense that a teacher who has a good procedural knowledge does not necessarily have good conceptual knowledge, and vice versa.

This finding may be due to the fact that procedural knowledge is independent of conceptual knowledge in mathematics, as there are many mathematical procedural processes mastered without having a good knowledge of how and why he has made these procedures. This was underlined by Baker and Czarnocha (2002) when they considered conceptual knowledge independent of individual's ability to apply procedural knowledge. It was again echoed by Vygotsky (1986) who believes that the development of individual's conceptual knowledge takes place through the reflection of pre-existing conceptual knowledge independently of the reflection resulting from repeatition of procedures and algorithms.

A review of the pedagogics literature revealed that this finding is inconsistent with the findings of some previous studies that found a correlation between math teachers' conceptual knowledge and their procedural knowledge (see Mahir, 2009; and Star, 2002).

Study Recommendations and Proposals:

Based on its findings, the study recommends the following:

- The necessity for mathematics teachers to use teaching methods and techniques focusing on conceptual knowledge in teaching rational numbers.
- The necessity for mathematics teachers to start focusing on conceptual knowledge regarding rational numbers in the primary cycle.
- The necessity for those concerned with education in general, and curricula and methods of teaching mathematics in particular, to focus on conceptual knowledge, through:
    - Conducting training sessions for both pre-service and in-service mathematics teachers regarding the use of teaching methods focusing on conveying conceptual knowledge to students.
    - Including conceptual knowledge evaluating methods, in addition to procedural knowledge evaluation, within the content of mathematics curriculum.
- The necessity for faculty members of curriculum and teaching methods at faculties of Pedagogic and Educational Sciences to develop mathematics teacher preparation programs through:
    - applying teaching methods that focus on conceptual knowledge in teaching, and including them in these programs.
    - Training math students during their Practice Teaching on focusing on conceptual knowledge in both teaching and evaluation processes.
- Conducting further studies designed to reveal mathematics teachers' conceptual and procedural knowledge regarding rational numbers.

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