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Secondary School Students' Misconceptions about Fractals

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Abstract

The purpose of this study is to determine the secondary school students' misconceptions about fractals. Three dimensions of students' misconceptions are examined: misconceptions on the definition of fractals, misconceptions of the recognition of fractals and errors in drawing fractal shapes. The study was conducted with 100 secondary school students (grade 8 and grade 10) with the use of an open-ended test consisting three questions prepared through literature review and Turkish mathematics and geometry curriculums (grade 8 and grade 10). The findings showed that students hold misunderstandings about formal definition of fractal. They had some problems in the drawing fractals although students could recognize intuitively the given shape as fractal or not.

Keywords: Fractal geometry, students' understandings, learning fractals, geometry curriculum, misconceptions

1. Introduction

Fractal geometry is a considerably new subject for mathematics and mathematics education. In recent years, fractal concepts started to be a part of mathematics curriculum (Grade 5-12 Virginia Mathematics Curriculum, 2009; Grade 8-12 Turkish mathematics curriculum, 2008) with the reforms in mathematics education. To reveal students' knowledge and understandings about fractal subjects in the current mathematics curriculum is important in determining possible difficulties of the subject and overcoming the difficulties.

It is known that at some grades, students experience conceptual difficulties about fractals (Bowers, 1991; Langille, 1996; Komorek, et al., 2001; Karakuş, 2011). Especially fractal concept, self-similarity and dimension are top subjects that students have difficulty in learning.

1.1. Students' Understanding of Fractals

Studies about including fractal geometry into the current mathematics curriculum were started after 1990. In the studies, teachers' learning environments related to fractal concept were frequently formed (Thomas, 1989; Goldenberg, 1991; Coes III, 1993; Vacc, 1999; Naylor, 1999; Lornell & Westerberg, 1999; Bolte, 2002; Barton, 2003; Devaney, 2004; Fraboni & Moller, 2008). However, there is little knowledge of how students would learn these subjects, what kind of knowledge and understandings they have and what kind of difficulties they have. Studies that focused on the teaching and learning of the fractal concepts reported that not only secondary school students, but also prospective teachers, experience difficulties in understanding fractals. Bowers (1991) conducted a study about teaching fractal concept with three 12th grade students.

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In this study, Bowers examined students' difficulties in forming the meaning of fractal concept, their understandings related to fractal concept and in which environment this concept can be taught. Bowers (1991) determined that students have difficulty in three subjects in learning of fractals.

The first is difficulties in learning fractal dimension, the second is difficulties in determining the scaling factor in the self- similar parts and the third is that the construction of a fractal is not obvious. Bowers especially stated that students' difficulties in determining the differences between the process of building the object and the object itself and difficulties in understanding the relation between iterative process and the definition of the fractal presented obstacles to the students understanding of fractals. Also, Bowers claimed that another reason of students' difficulties in understanding fractals is epistemological obstacles about fractals. Similarly, Langille (1996) conducted a study about integration of fractal geometry into the 12^{th} grade mathematics curriculum. He determined that students have difficulty in understanding dimension, self-similarity dimension and in determining characteristics of fractals. Komorek et al (2001) investigated that whether teaching fractal geometry and chaos theory in science education is a worthy and whether 15-17 students can understand these subjects.

The results of the study revealed that students can determine an objects' self-similarity intuitively, but they have difficulty in defining self-similarity in mathematically and in determining magnification factor (scaling ratio). Karakuş (2011) determined that pre-service teachers can generally decide whether a given shape is a fractal or not, but they have difficulty in finding patterns about perimeter and area of fractals and learning of fractal dimension. He stated that pre-service teachers rather have difficulty in determining self-similar parts number and magnification factor in calculating the fractal dimension of an object. Moreover, self-similarity of natural objects is another obstacle for pre-service teachers in forming self-similarity concepts in their minds.

The most important reason of this was stated as a result of the definition used in the explanation of self-similarity concept. Especially the definition used in deciding about self-similarity by comparing any part of the object and whole object is inadequate. In addition Karakuş (2013) examined how the students understand fractals depending on age. He found that in all grades, students had lack of knowledge and misunderstandings about fractals. Moreover, students can identify and determine the fractals, but when the grade level increased, this success decreases.

1.2. Fractals in Turkish Educational Setting

In Turkey, students' first encounter with the concept of fractals takes place during the last year of Grade 8 mathematics education. Grade (6-8) Mathematics Education curriculum includes a goal about fractals *"to build patterns from line, polygon and circle models, to draw them and to determine fractals from these patterns"* (MEB, 2008a). The goal was prepared in terms of building fractal patterns by using figures in Euclid geometry or deciding whether given patterns are fractals or not. When the textbooks were examined, the definition of fractal is written as "the *patterns which were built proportionally with the magnified and reduced of a shape"* (Aydın & Beşer, 2008; Cinkol, 2010; MEB, 2008b). The definition emphasizes important properties of fractals as iteration and self-similarity. However, the activities in which patterns in fractal shapes or mathematical patterns such as fractals' perimeter, area and volume are found and generalized are not included in this grade. So, there are activities such as drawing fractals or finding fractals from given shapes. Moreover, there are natural fractal examples such as fern, branches of tree in the 8^{th} grade mathematics curriculum.

When Geometry curriculum (9-12 grades) was examined, it is seen that there are two goals about fractals in the grade 10 geometry curriculum (MEB, 2010). In the first goal, the following statement is included *"To build fractals with segments, to explain them and to compute the length of the fractal in a particular step".* In the second goal, the following statement is included *"To build fractals with triangles, to explain them and to compute the area of fractal image in a particular step".* In this grade, fractal shapes are formed by using Euclid shapes and there are activities related to finding patterns in these shapes.

In conclusion, in Turkish mathematics and geometry curriculums, fractals are studied as follows; recognition of fractals in the grade 8, formation of fractal shapes and determination of patterns of these shapes in the grade.

1.3. The Purpose and Research Questions of the Study

As we mentioned earlier, few studies in educational research reported that both students and pre-service teachers posses misconceptions about fractals. The overall aim of this study is to provide an account of Turkish secondary (Grade 8 and 10) students' misconceptions on the concept of fractals.

2. Method

2.1. Participants

The population sample for this study consists of 63 (28 boys and 35 girls) secondary school students. Thirty-four students (13-14 years old) were in Grade 8 (17 boys and 17 girls) and twenty-nine students (15-16 years old) were in Grade 10 (11 boys and 18 girls). Our aim in selecting the students from two different grades was to determine the effect of the mathematics curriculum on the students' understandings about fractals. For the purpose of all of the students have knowledge about fractals; the study was conducted at the end of 2010 spring semester.

2.2. Data Collection

In this study, an open-ended test consisting three parts was used as the data collection tool. In the first part of the test, students were asked to define the formal definition of fractals. In the second part of the test, students were asked to recognize whether the given geometric shapes are fractals or not. In the third part of the test, students were asked to draw a fractal shape.

The questions in the form were prepared by considering the goals of the 8^{th} and 10^{th} grade mathematics and geometry curriculums. Questions were revised according to the opinions of the experts and questions which measure the same property were taken out of the form. The categories of questions are shown in Table 1.

Table 1: Categories of Questions

Categories	Questions	Aims
Defining fractals	We asked the students to write the definition of fractals	To determine students' knowledge about the fractals
Recognizing shapes as fractal or not	We asked the students to determine which class (fractal or not) the nine shapes belonged to Classify the following shapes as fractal or not and explain why?	To determine whether students recognize a fractal shape or not
Drawing fractal shapes	Draw a fractal shape and explain why it is a fractal	To determine students' fractal image in their mind

2.3. Data Analysis

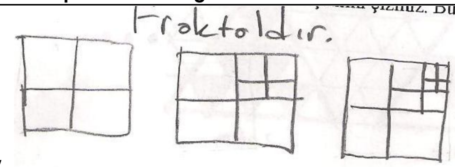
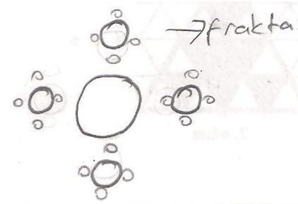
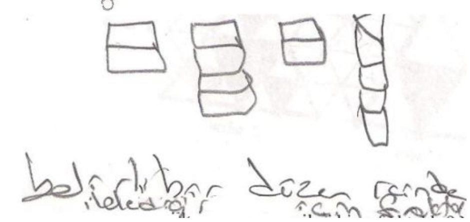
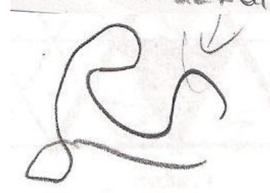
The open-ended test was given to the students and asked them to answer the test. The data obtained from the first part were categorized as true, false and incomplete. The criterias of categories for first section are shown in Table 2.

Table 2: Criterias of Categories for the First Section

Categories	Criterias
True	If students directly state the definition in the textbooks "fractals are defined as the patterns which were built proportionally with the magnified or reduced of a shape" or define as "consistently magnifying and reducing regular shapes" by emphasizing two main properties of fractals "iteration" and "self-similarity".
False	If students state the definition in the textbooks wrongly or they don't include the terms that imply "iteration" and "self-similarity" properties in the definition. For example; "fractals are only patterns" or "fractals are repeated shapes"
Incomplete	If students don't define or they define in an irrelevant way

In the second part, students were given 9 different pictures and asked them categorize these pictures as fractal or not. Data were categorized as fractal, not fractal and blank. Data obtained from the third part of the test were categorized as correct, incorrect and incomplete drawings. The criterias of categories for the third section are shown in Table 3.

Table 3: Criteria of Categories for the Third Section

Categories	Criterias	Sample drawing
Correct	According to the initiator, students draw shapes appropriate to the rule in the generator (the first iteration), which defines formation rule of the fractal, correctly in every iteration; students apply magnifying and reducing in a certain factor to drawing shapes with considering defined iteration rule by using self-similarity property (that is, when any piece of the shape is magnified, the shape looks similar to the whole)	 or 
Incorrect	Students define incorrect iteration rule according to the initiator or apply the defined rule wrongly in every iteration; students aren't careful about self-similarity property. Students can draw magnifying or reducing shapes, but these shapes don't have self-similarity.	 or 
Incomplete	Students draw irrelevant shapes or don't draw any shape.	

3. Results

3.1. Definition of Fractals

The aim of the first question used in the open-ended test was to determine students' definitions of fractals. With this aim, we asked the students to give definitions for fractals. We used the following definition which exists in the Turkish mathematics curriculum in the 8th grade as the criteria to determine whether an answer is true: *"fractals are defined as the patterns which were built proportionally with the magnified or reduced of a shape"*. Responses of the students were categorized as correct, incorrect and incomplete answers and data obtained were given in the Table 4.

Table 4: Frequencies and Percentages of Correct, Incorrect, and Incomplete Answers

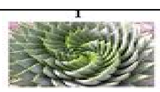
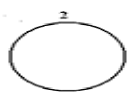
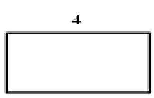
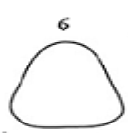
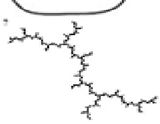
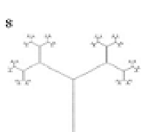
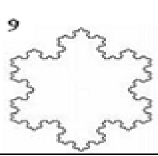
Grade	True		False		Incomplete	
	f	%	f	%	f	%
8	22	64,7	5	14,7	7	20,6
10	13	44,8	9	31	7	24,2

As seen from Table 4, the percentage of the 8th grade students' correct fractal definition is considerably high compared to the percentage of the 10th grade students (64.7%). However, the percentage of the 10th grade students' false fractal definition is higher than the 8th grade. It is observed that students who define fractal concept incorrectly center on three definitions. The first of these definitions is *"fractals are regular patterns"*. As seen from this definition, some students in all grade levels didn't know the difference between pattern and fractal exactly. Therefore, they think that all patterns are also fractals. Another incorrect and common definition is that *"repeated shapes are fractals"*. Some students think that if a shape has an iteration rule, it must be a fractal. The last fractal definition categorized as incorrect is that students state fractals only as reducing shapes one within the other. The reason of this situation may be that students often study with fractals reducing one within the other especially in the activities of textbooks.

3.2. Recognizing Shapes as Fractals or Not

In this section, nine different shapes were presented to students whom were asked to determine which class these shapes belonged to. Percentages of the results are shown in Table 5. In the table, grey cells indicate the correct class for each shape.

Table 5: Percentages of Correct and Incorrect Answers

Shapes	Grades	Fractal		Not Fractal		Blank	
		f	%	f	%	f	%
	8	24	70,6	10	29,4	0	0
	10	28	96,6	1	3,4	0	0
	8	10	29,4	24	70,6	0	0
	10	1	3,4	28	96,6	0	0
	8	26	76,5	8	23,5	0	0
	10	17	58,6	12	41,4	0	0
	8	9	26,5	25	73,5	0	0
	10	0	0	29	100	0	0
	8	34	100	0	0	0	0
	10	29	100	0	0	0	0
	8	10	29,4	23	67,6	1	3
	10	1	3,4	28	96,6	0	0
	8	25	73,5	9	26,5	0	0
	10	21	72,5	7	24,1	1	3,4
	8	34	100	0	0	0	0
	10	26	89,7	3	10,3	0	0
	8	19	55,9	15	44,1	0	0
	10	23	79,3	6	20,7	0	0

As seen in Table 5, the majority of the students in all grade levels classify correctly the first shape, spiral plant, as fractal. In this classification, it can be said that the 10^{th} grade students have higher success than other grade students. When the grade levels increase, the 10^{th} grade students have better performance than the 8^{th} grade students. For the first shape, only 3,4% of the 10^{th} grade students answer incorrectly.

The majority of the students stated correctly the second shape, circle, as not fractal. Similar to the first shape, as the grade levels increase, the percentage of correct answers increases (respectively 70.6%, and 96.6%). Moreover, it is an interesting result that 29.4% of the 8^{th} grade students classify circle as fractal. In the third shape, tree, more than half of the students in all grades answered correctly. However, the 8^{th} grade students have higher success than the 10^{th} grade students. Almost half of the 10^{th} grade students accepted that a tree was not a fractal.

Students in all grades have a great success in terms of classifying the fourth shape, rectangle, as not fractal. As the grade levels increase, the percentage of correct answers increases and all of the 10^{th} grade students stated correctly the fourth shape as not fractal. The interesting point is that 26.5% of the 8^{th} grade students stated that square is a fractal. All of the students in the 8^{th} and 10^{th} grade classified correctly the fifth shape, fern, as fractal.

In the classification of the sixth shape, the majority of the students in all grades classified correctly (respectively 67.6%, and 96.6%). Also, as the grade levels increase, correct answers as not fractal increase. In the seventh shape, more than of the students in every grade classified correctly as fractal. In this classification, the percentages of success of the 8^{th} and 10^{th} grade levels are close.

In the eight shapes, although, most of the students classified correctly as fractal, the 8^{th} grade students are more successful than the 10^{th} grade students. More than half of the students classified correctly the ninth shape, snowflake. As the grade levels increase, the number of correct answers increases.

As seen in Table 5, it can be said that more than half of the students in all grades classified the given shapes correctly as fractal and not fractal. As expected, while the grade levels increase, the number of the correct classification increases.

However, in the classification of the third shape, the 8^{th} grade students were the most successful.

It can be seen from the writings of students that when students in all grade levels classify the given shapes as fractal and not fractal, they mostly focus on magnifying and reducing of the shapes with a particular rate. Moreover, some students in every grade level consider whether shapes follow each other or not. To focus on only this property in the classification as fractal causes to classify the third shape and the seventh shape as not a fractal. Also, it is determined that students think about iteration of shapes in a pattern in the classification of shapes as fractal. This situation causes to classify the third, seventh and ninth shapes incorrectly as not fractal.

3.3. Drawing Fractal Shapes

In this part, students were asked to draw a fractal shape. The purpose is to determine what kinds of shapes are accepted as fractal by students according to the drawings of the students. The drawings of the students are classified as correct, incorrect and incomplete drawings and the data are given in the Table 6.

Table 6: Percentages and Frequencies of Students' Correct, Incorrect and Incomplete Drawings

Grades	Correct		Incorrect		Incomplete	
	f	%	f	%	f	%
8	24	70,6	10	29,4	0	0
10	15	51,7	13	44,8	1	3,5

As seen from Table 6, the percentage of the correct fractal drawings of the 8th grade students are higher than percentage of the 10th grade students. It is observed that students in every grade who draw shapes incorrectly focus on two drawings with the same properties.

The first one is drawings that students can't apply the rule in every iteration step correctly. For example; Ayşe, one of the 8th grade students, drew the following fractal shape (Figure 1).

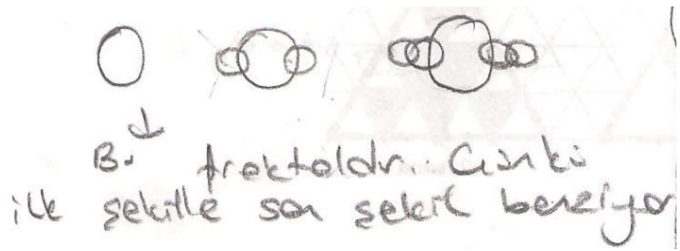


Figure 1: Drawing of Ayşe's Fractal Shape

In the drawing of Ayşe, the first step is to determine fractal generator by drawing two small circles to left and right of the first circle. However, in the next step, she drew one circle to the left of the previous left circle and one circle to the right of the previous right circle instead of the generator fractal. In the previous step (the generator step), she added reduced circles to both right and left.

Similarly, the fractal drawing of Ahmet who is one of the 10th grade students is given below (Figure 2).

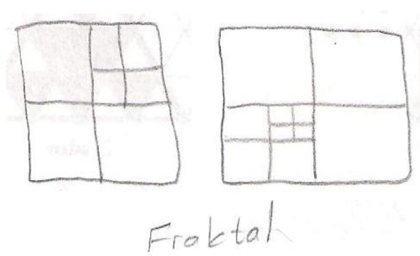


Figure 2: Drawing of Ahmet's Fractal Shape

Ahmet used squares when building fractal and divided square into 4 equal parts. Then, he divided the right upper square into 4 equal parts and defined the generator. However, in the next step, he changed the rule and applied the same operation for the left down square.

Metin who is one of the 10th grade students drew in a similar shape (Figure 3).

Figure 3: Drawing of Metin's Fractal Shape

Metin determined the generator by dividing a square into 2 horizontally and then dividing the upper part into 4. But, in the next step, he divided only the left upper part into 4 and formed a fractal.

The second one is that students draw regular shapes as fractals since they think that shapes with a pattern are also fractals. Students who have this idea accept all repeated shapes with a rule as fractal.

For example; drawing of Melis who is one of the 8th grade students is given below (Figure 4).

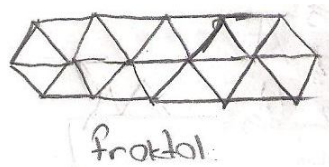


Figure 4: Drawing of Melis's Fractal Shape

Melis formed a pattern shape and emphasized this as a fractal. Similarly, Umut who is one of the 10th grade students emphasized that shapes in an order are fractals (Figure 5).

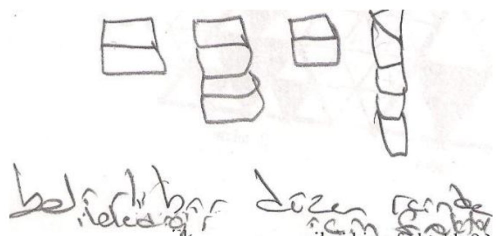


Figure 5: Drawing of Umut's Fractal Shape

However, drawing of Umut is a shape constructed by ordering particular size squares consecutively. Similarly, İlayda who is one of the 10th grade students stated that shapes in an order are fractal when explaining drawing (Figure 6).

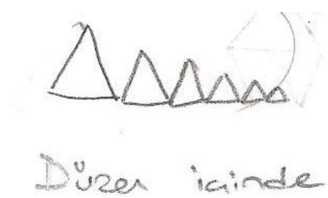


Figure 6: Drawing of İlayda's Fractal Shape

When drawing of İlayda were examined, it can be said that unlike Umut, she was careful about putting shapes in an order and repeating them as lessening in a specific rate.

4. Discussion and Conclusion

In this study, students' knowledge and misconceptions on fractals were examined. The results show that the 8th grade students define fractal more correctly than the 10th grade students. The reason of this may be that there is a definition of the fractal only in the 8th grade textbooks. In the 10th grade, there are some activities such as finding patterns related to perimeter and area of fractals and drawing fractal shapes. There aren't so much activities and explanation about what fractal is. Moreover, it was seen that there are similar incorrect definitions in every grade level. In these definitions, fractals are accepted as regular patterns or emphasized about the iteration property. One of the reasons of incorrect definitions may be that the generation of a fractal isn't exactly clear as Bowers (1991) stated, since students may not understand in which step of transformation fractal is formed when they were transforming a familiar geometric shape into another shape by iterating in an order. This situation may cause students to have some conceptions; if there is only a pattern in the shape, it is a fractal or if a shape is repeated, it is a fractal. Also some epistemological obstacles related to fractal concept cause students to draw incorrect shape. Epistemological obstacles may be that fractal concept has an infinite structure and to understand a shape with infinite structure is difficult.

The majority of the students in every grade level could determine the fractals from the given shapes correctly. This situation shows that students can recognize the fractal shapes intuitively as stated in the studies of Karakuş (2011; 2013) and Komorek et al (2001).

Moreover, as the grade levels increase, the students' success in determining fractals from the given shapes increases. It can be said that the present mathematics and geometry curriculums have an effect on this. Also, in classification of the third shape, it is determined that the 8^{th} grade students are more successful than others. One of the reasons may be that the 8^{th} grade students are more familiar with the shapes tree, fern and fractal tree. Most of these shapes are described to the eighth grade students when they firstly study with the fractal concept. In other grade level, the mathematical bases of these shapes are studied. Another reason may be the deficiencies in the definition of fractal in the curriculum. The given definition is not enough especially in natural fractal shapes. Karakuş (2011) stated that pre-service teachers have difficulty in determining the self-similarity of natural objects.

The reason of this is the definition of self-similarity which is formed by comparing the part of the object and the whole of the object is insufficient for natural fractals. The definition of fractal in the present textbooks emphasizes the similarity between the part of the object and the whole of the object in determining an object as fractal. So, the results of this study are parallel with the study of Karakuş (2011; 2013).

When the fractal drawings of students were examined, it is determined that students in every grade level generally have correct fractal drawings. However, as the grade levels increase, the rate of correct drawings decreases. The reason of this may be that there are more activities related to fractal drawings in the 8^{th} grade mathematics curriculum than other grade. However, when incorrect drawings were examined, two common problems were found for every grade level. The first one is that students can't define the rule in the pattern well while drawing the fractal shape or they aren't careful about the rule. The second one is that students accept all patterns repeated in an order as fractals. This situation shows that students can't get the difference between fractal and pattern. Since students can't define the rule in the pattern well while drawing the fractal shape or they aren't careful about the rule, they don't understand exactly how fractal is formed. This finding is in coherence with the study of Bowers (1991).

Educational Implication

The results of this study showed that students generally know the definition of the fractal, recognize fractals intuitively and draw simple fractal shapes. However, the results also show that there are common defining and drawings fractals in every grade level. Therefore, some changes to the content of the courses must be made. Especially given definition of fractal is deficient in determination of some fractals such as fern, broccoli, and coastline. Students mostly determine fractal shapes intuitively or by memorization. Also, they have difficulty in differentiation of fractal and pattern. Moreover, the mistakes of students in drawing fractal shapes show that they have deficiencies in understanding how fractal is built. Therefore, to have a formal way in formation of fractal such as initiator-generator-iteration may be effective in overcoming this difficulty. We think that to pay attention to especially the generator, which provides the formation of fractal, may support success in both defining and understanding the rule and applying the rule in the iteration steps.

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