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Pre-Service Elementary School Teachers' Level of Mathematical Thinking and their Attitudes toward Mathematics

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Abstract

The focus of this study is to examine the pre-service elementary school teachers' mathematical thinking and their attitudes towards mathematics. The researcher also examined the correlation between the students' mathematical thinking and their attitudes toward mathematics. The study sample consisted of eighty female students. The researcher employed two types of instruments: the Attitudes towards Mathematics Inventory (ATMI) and Mathematical Thinking Test (MTT). The research results revealed 1) there was a positive correlation between students' mathematical thinking and their attitudes toward mathematics. 2) That the average students in mathematical thinking test is moderate; and students' performance is the best in the Modeling and Induction aspects. In addition, the average is weak in Mathematical Proof and Generalization. 3) The students' performance in the mathematical thinking test in some aspects varies depending on the specialization in the secondary school in favor of students of the scientific stream. 4) The performance of students on mathematical thinking test shows the growth in some aspects of mathematical thinking while moving from one academic year to another. 5) The students have positive attitudes toward mathematics.

Keywords: Thinking, Mathematical Thinking, Attitudes toward mathematics

1. Introduction

In light of the advancement of knowledge, technological development, and rapid change in our lives today, there is a need to prepare students to keep up with the social, economic, scientific and technological fields. This can be achieved by changes that become paramount in giving students the skills to think because teaching thinking provides individuals with the tools needed to deal effectively with any type of information or variables that they may encounter in the future.

The development of thinking has received care and attention from educational administration. Giving students facts and filling their minds with information is no longer the goal of the educational process; rather, it is the development of thinking and teaching students how to learn and how to think.

Researchers confirm that it is not enough to understand the learning process, as a cognitive process, but it must go beyond that to understand the factors affecting the learning process, such as attitudes, beliefs and other factors to be able to improve the learning process (Maasz & Schlöglmann, 2006). Mathematics, in particular, is a product of and a potential of the human mind. People in various cultures use mathematics for conducting everyday life matters. In general, learning mathematics is one of the important things that we need as human beings.

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NCTM Standards emphasize that mathematics is a dynamic subject that seeks to understand the patterns that permeate the entire world around us and within our mind. The language of mathematics is based on the rules that must be learned by the individual; it is therefore important to stimulate students' motivation to be able to use the language of mathematics to express things. This change and development emphasize the need to bring changes in both the content and style of the curriculum (NCTM, 2000).Mathematical thinking works as a driving force to elicit knowledge and skills by helping the individual to understand and grasp the necessary knowledge and skills to solve problems. It should be seen as the driving force of behavior and knowledge (Katagiri, 2004). Mathematical thinking is a specialized function distinguished from generalized thinking and the continuous cyclical process of cognition (Argyle, 2012).

Mathematical thinking is located in the mind as the mental treatment of mental objects. What is written or said is the expression of those mental processes (Maasz & Schloeglmann, 2006). Katagiri (2004) considers mathematical thinking as the "scholastic ability" which should be cultivated in math courses. Learning mathematics as a process of knowledge includes one of several key aspects of learning.

The tools of mathematics are: abstraction, symbolic, representation, and symbolic manipulation. However, being trained in the use of these tools without thinking mathematically does not mean that one is mathematician or a scholar in this field. Learning to think mathematically means developing a mathematical point of view, valuing the processes of mathematization and abstraction, and having the predilection to apply them.(Schoenfeld, 1992). Most scientists and researchers emphasize that conjecturing, reasoning and proving, abstraction, generalization and specialization are important aspects of math thinking (Breen & O'Shea, 2010). Mathematical thinking is divided into three categories: (1) thinking related to mathematical methods; (2) thinking related to mathematical content; and (3) mathematical attitudes, which act as driving force for the first two categories (Katagiri, 2004). The person who thinks mathematically has a special way of looking at the world, and a special way to represent and analyze it (Schoenfeld, 1992). Mathematical thinking is applied during math activities and therefore is closely linked to the contents and methods of arithmetic and mathematics. A multitude of different methods and mathematical contents are used when arithmetic or mathematics is applied to perform mathematical activities (Katagiri, 2004). To perform teaching for understanding, teachers need to manage problem-solving situations in classroom; understand the students' thinking and identify the mathematical key aspects in the students' thinking during problem solving (Fernández et al., 2013).

In the field of educational mathematics, it is vital that the learner acquires a wide range of thinking skills to develop his/her mathematical abilities. The development and completion of new mathematics content are no longer considered the only factors that enhance a learner's mathematical abilities. Most courses and programs focus on mathematical content, and students are expected to pick up the mathematical habits or mathematical mentality according to their needs. This is useful and effective to a number of students. Breen & O'Shea (2010) emphasize that the most effective ways that mathematicians use to support and assist students are to assign a large number of tasks that students need in order to develop their skills in mathematical thinking. Common wisdom states that mathematical thinking can be developed through practical and applied contributions that help students reach the ability to express and communicate mathematical representation (Van, 2006).

Many researchers believe that mathematics is linked to a variety of factors such as attitudes, beliefs, and motivation; this relationship is relatively stable, but it is susceptible to change. In addition, variables such as sex and achievement in mathematics are associated with additional variables such as attitudes towards mathematics, beliefs and motivation (Hannula, 2006).

Motivation is considered as one of the components of attitudes. The term motivation is used in a simple and narrow way in educational studies, but it is used extensively in the field of educational psychology. It is not easy to say that motivation leads to better achievement. Attitudes towards mathematics and its various components can be considered as a tendency to prefer mathematics or not prefer it.

Implementing educational objectives in the classroom is influenced by various factors including: students' beliefs about themselves, their beliefs about mathematics and the learning environment. Therefore, to change students' motivation and attitudes towards mathematics, it is necessary to determine the goals of teaching and boost the student's beliefs and attitudes. (Hannula, 2006). Attitudes can be considered a trend toward a particular style of behavior or tendency toward a certain type of emotion in a specific field or domain such as mathematics (Maasz & Schlöglmann, 2006). Attitudes towards mathematics are the beliefs of individuals and their perceptions about mathematics and its role and importance in their lives as well as their abilities in the field of mathematics (Sanci, 2014).

Positive attitudes towards mathematics may increase the tendency of the individual to continue to learn about mathematics, to regularly take advanced courses in mathematics and to choose a career or work that requires mathematics or mathematics related skills. (Sanci, 2014). There is great interest in students' performance, achievement, mistakes and scores in mathematics, but students' attitudes towards mathematics have not been given due attention (Tapia & Marsh, 2000). Identifying and studying students' attitudes towards mathematics are an important issue because success or failure in mathematics is affected by personal attitudes, beliefs, and other attitudinal factors (Sanci, 2014). Attitudes are considered the most important factor for success; therefore, understanding students' attitudes towards mathematics is important for educators in the field of mathematics. It provides them with opportunities to identify negative and positive attitudes towards the subject. Furthermore, it enables educators to design a suitable classroom environment and employ appropriate, effective and useful teaching strategies.

We need to learn a lot about how attitudes are formed and changed to prevent failure in mathematics. Negative attitudes toward mathematics, low self-confidence and low motivation influence career options and guide away from the field of mathematics (Asante, 2012). Change in attitude towards mathematics and improving achievement and performance in mathematics is considered a plan for future job development (Tapia & Marsh, 2000).Negative attitudes towards mathematics may weaken students' performance in math classes, reduce career options, and promote unwanted behavior toward any activities that include mathematical operations, whether be it simple arithmetic operations or complex problems. Research in the area of attitudes towards mathematics confirms the importance of developing positive behaviors towards mathematics at a young age and links positive attitudes towards mathematics with good performance in mathematics (Sanci, 2014).

Ernest (1988) explains that attitudes towards mathematics are less important than it is customary believed by researchers and confirms that mathematical knowledge lies behind the weakness or strength of the attitudes. He also confirms that the attitude towards mathematics differs from the attitude towards the teaching of mathematics and that the last is considered the most important for mathematics teachers.

Teachers' knowledge of mathematical content is not enough to make them good teachers. Beliefs about mathematics as well as learning and teaching methods affect not only the ways teachers use in the teaching mathematics, but they also affect the learning methods of pre-service mathematical teachers (Philipp, 2008).

Students in Jordan do not have enough mathematical skills, and this weakness is the outcome of the teaching methods prevalent in our schools. To prepare students in a rapidly changing world, it is necessary to find educational teaching strategies that are not based on the transfer of knowledge and the teaching of this knowledge. Rather, the system has to help students in the development of their own abilities, and the development of their thinking, so they can face challenges, and help their community achieve desirable changes. (Abed & Asha, 2009). As for the studies that deal with aspects of mathematical thinking, some have confirmed that there is an improvement in the mathematical thinking ability through the progress of students, from high school to university, and from year to year (Abed & Abu Zainah, 2012; Abu-El Huda, 1985).

In terms of studies that have investigated the reality of mathematical thinking, and its impact on school mathematics curriculum, the results indicate that school mathematics textbooks provide content and mathematical concepts, algorithm, generalizations, and mathematical skills, without taking into consideration different learning styles and math thinking skills (Nejem, 2004). Studies of teachers' orientation towards the development of students' mathematical thinking have shown that teachers' efforts towards developing students mathematical thinking have been limited (AbuAl-Haija, 2004).Some studies have also addressed the impact of training programs and specific teaching strategies on developing mathematical thinking (Al-Khateeb and Ababneh, 2011;Abed& Asha, 2009;Nejem, 2012; Eleela, 2012). Other studies have focused on the relationship between mathematical thinking and students' attitudes towards mathematics, and the impact of training programs on students' attitudes towards mathematics, and the development of mathematical thinking skills. The findings of these studies have confirmed the existence of a strong positive relationship between achievement in mathematics and mathematical thinking (Al-Khateeb, 2004; Kargar, et al., 2010).

In addition, some studies have dealt with the nature of the relationship between aspects of mathematical thinking and mathematical problem solving. Limjap (2011) focuses on the importance of solving math problems by introducing activities in mathematics classes and focusing on finding solutions to problems that are not based on standard algorithms. Fernández and his colleagues (2013) deal with the characteristics of pre-service teachers and their mathematical thinking aspects. Their study results show that pre-service teachers find it difficult to identify the relevant aspects of mathematical thinking and their relationship to the progress of students from additive thinking to multiplicative thinking. Other which, have discussed attitudes towards learning and teaching mathematics confirm the presence of a positive relationship between experiences in mathematics and attitudes towards mathematics (Jong & Hodges, 2013; Sanci, 2014; Meng, 2012; Afari, 2013).Other studies have also tackled the relationship between mathematical thinking and student learning styles (Asha & Al-Absi, 2013).

One of the primary concerns of research in mathematics education is to identify various aspects of mathematical thinking, like mathematical learning, knowing, perception, meaning, remembering, emotion and others (Barwell, 2009).

This study tries to fill the gap by disclosing mathematical thinking aspects among students, and their relationship to their attitudes toward mathematics. This knowledge may contribute to developing programs and materials to support students' mathematical thinking. It will also contribute to improving their attitudes towards mathematics.

The conversant with the mathematics curricula in Jordan are aware of the problems facing teachers and students in teaching and learning mathematics. This study is an attempt to draw the attention of mathematics teachers and curricula developers to increase attention to mathematical thinking and emphasize its development through the teaching of mathematics curricula and content, which should include mathematical problems. This is in line with the recommendations of both the American National Council of Mathematics Teachers (NCTM) and the Ministry of Education in Jordan. These efforts are consistent with global trends towards the development of education and teaching methods to support students' learning and to enable them to employ mathematical knowledge and skills in practical life.

This goal becomes more important for pre-service teachers preparing to become teachers for the younger generations in a few years' time. This study focuses on the development of mathematical thinking in pre-service students at the University of Petra. It also explores the relationship between their mathematical thinking and their attitudes to mathematics.

1.2 Study Questions

1- What is the level of mathematical thinking for the pre-service students?
2- Are there differences in the pre-service students' level of math thinking based on specialization in high school?
3- Are there differences in the pre-service students' level of math thinking based on the academic year level?

4- What are the pre-service students' attitudes towards mathematics in overall The Attitudes towards Mathematics Inventory(ATMI)and in its four dimensions?
5- Are there differences in the pre-service students' attitudes towards mathematics based on specialization in high school and academic year level?
6- Is there a correlation between the pre-service students' mathematics thinking and their attitudes towards mathematics?

1.3 Operational Definitions

1.3.1 Mathematical Thinking

The ability to build hypotheses and draw conclusions using mathematical properties, relationships and links. It identifies the following aspects: Induction, Generalization, Symbolism, Deduction, Logical Thinking, Estimate, Modeling and Mathematical Proof. It is measured by the mark obtained by the pre-service student in mathematical thinking test developed by the researcher based on the test prepared by (Al- Khateeb, 2004).

1.3.2 Attitudes Toward Mathematics

Attitudes are the positive or negative feelings about a subject, a person, a situation or a thought. They may also be considered as a trend towards certain types of behavior, or the tendency toward certain types of emotional or feelings in particular domains (Maasz & Schloglmann, 2009). Attitudes toward mathematics in this study are the pre-service teacher's feelings towards mathematics which are formed by his/her experiences; the extent of enjoying mathematics, assessing the value and the scientific and practical importance of mathematics as well as the difficulty faced when studying mathematics. It is measured by the degree of acceptance or rejection by the pre-service students to inventory statements prepared for this study.

2. Methodology

2.1 Participants

Eighty pre-services students from the Department of Educational Sciences at University of Petra enrolled in the first and second semesters of the academic year (2013/2014) participated in this study. Table (1) shows the student numbers and their percentages in the four years of study. It also shows the different specializations in high school (Sciences/ Arts/ IT/ Others).

Table (1): Student Numbers and Their Percentages in the Four Years of Study as well as Their Specializations in High School

	First-year		Second-year		Third-year		Fourth-year		Total	
	No	%	**No**	%	**No**	%	**No**	%	**No**	%
Sciences	2	13.3%	5	33.3%	4	26.7%	4	26.7%	15	18.8%
Arts	7	22.6%	7	22.6%	16	51.6%	1	3.2%	31	38.8%
IT	12	40%	12	40%	4	13.3%	2	6.7%	30	37.5%
Others	0	0%	0	0%	3	75%	1	25%	4	5%
Total	21	26.3%	24	30%	27	33.8%	8	10%	80	100%

2.2Instruments (data collection tools)

2.2.1 Mathematical Thinking Test (MTT)

To determine the levels of student's mathematical thinking, the researcher used an adjusted form of a test prepared by Al-Khateeb (2004). These adjustments were carried out by the researcher. The test consists of eight aspects assessed by (40) items:

Induction: Reaching the general provisions or results based on particular cases or particulars of the general situations or moving from specific instances to general situation.

Deduction: Accessing private situations depending on the result of a general principle or hypotheses.
Generalization: Note formulation, or an expression that is reached by induction.
Symbolism: Writing or expression using symbols and abstractions of words or phrases.
Logical Thinking: Intellectual ability enabling the individual to move intended known to the unknown, guided by the rules of logic and principles agreed.
Estimate: Guessing or conscious appreciation of the amount intuitively and without waiting for the result of the analysis.
Modeling: Mathematical representation of the elements and relationships in a perfect copy of a phenomenon or relationship.
Mathematics proof: Evidence or argument to show that validity of the phrase originates from the previous statements is true.

The reliability coefficient (Cronbach Alpha) has been calculated using a sample group of 50 students; the reliability value for (MTT) scale as a whole was (0.895), the reliability value for each factor is as follows: Induction (0.694), Generalization (0.742), Symbolism (0.635), Deduction (0.705), Logical Thinking (0.415), Estimate (0.513), Modeling (0.573) and Math Proof (0.459).

2.2.2 Attitudes towards Mathematics Inventory (ATMI)

The Attitudes towards Mathematics Inventory (ATMI) was designed to assess several dimensions of attitudes toward mathematics (Tapia & Marsh, 2004). The Inventory includes 40 items that assess enjoyment (10 items), motivation (5 items), self-confidence (15 items), and value of mathematics (10 items). These items were graded on a five-point Likert scale: (1 strongly disagrees, 2 disagree, 3 neutral, 4 agree, and 5 strongly agree). The researcher translated the instrument (ATMI) into Arabic. Additionally, the researcher had the translation reviewed by a group of professional reviewers (although it already possesses great validity and reliability in its original format) to ensure that it is suitable for the Jordanian context. The reliability coefficient (Cronbach Alpha) has been calculated using a sample group of 50 students. The reliability value for (ATMI) scale as a whole was (0.961). The number of items and reliability value for each factor is as follows: enjoyment (10 items, $\alpha = .918$), motivation (5 items, $\alpha = 0.787$), self-confidence (15 items, $\alpha = 0.937$) and value of Mathematics (10 items, $\alpha = 0.888$).

2.2.3 Limitations of the Study

The results of this study are limited in the light of the following factors:

1- The study was limited to students at the University of Petra.
2- The results were determined by characteristics of the scales used, and their ability to detect differences between students in the mathematical thinking test. The first test (MTT) was prepared to test the following thinking aspects: Induction, Generalization, Symbolism, Deduction, Logical Thinking, Estimate, Modeling and Math-Proof. The second test was The Attitudes towards Mathematics Inventory (ATMI).
3- All study subjects are females.

3. Results

First question: What is the level of mathematical thinking for the pre-service students?

The researcher has calculated the percentage means of the students' scores for each aspect of the mathematics thinking in each year and for all specializations. The results are shown in Table (2).

Table (2): Percentage Means of Students on the Overall Mathematical Thinking Test and its Eight Aspects Depending on the Variables: Specialization in High School and Academic Year-Level

Sciences	First-y	2	60	50.0	20.0	70.0	40.0	0.0	70.0	10.0	40.0
	-y	5	72	52.0	76.0	88.0	56.0	56.0	84.0	60.0	68.0
	Third-y	4	65	65.0	65.0	70.0	80.0	60.0	70.0	65.0	67.5
	Forth-y	4	60	40.0	65.0	65.0	80.0	70.0	95.0	60.0	66.9
	Total	15	65.3	52.0	62.7	74.7	66.7	53.3	81.3	54.7	63.8
Arts	First-y	7	77	8.6	37.1	45.7	62.9	40.0	80.0	31.4	47.9
	Second-y	7	82.9	40.0	57.1	74.3	54.3	60.0	82.9	37.1	61.1
	Third-y	16	76.3	37.5	61.3	52.5	71.3	47.5	63.8	41.3	56.4
	Forth-y	1	20	0.0	20.0	80.0	80.0	60.0	100.0	20.0	47.5
	Total	31	76.1	30.3	53.5	56.8	65.8	49.0	72.9	37.4	55.2
IT	First-y	12	65	23.3	30.0	41.7	65.0	41.7	68.3	43.3	47.3
	Second-y	12	60	31.7	45.0	60.0	41.7	48.3	68.3	36.7	49.0
	Third-y	4	60	35.0	30.0	70.0	70.0	45.0	70.0	30.0	51.3
	Forth-y	2	100	80.0	50.0	100.0	70.0	50.0	70.0	60.0	72.5
	Total	30	64.6	32.0	37.3	56.7	56.7	45.3	68.7	40.0	50.2
Others	First-y	0									
	Second-y	0									
	Third-y	3	80	40.0	60.0	60.0	53.3	60.0	73.3	40.0	58.3
	Forth-y	1	60	40.0	60.0	40.0	80.0	40.0	100.0	40.0	57.5
	Total	5	75	40.0	60.0	55.0	60.0	55.0	80.0	40.0	58.1
Total	First-y	21	68.6	21.0	31.4	45.7	61.9	37.1	72.4	36.2	46.8
	Second-y	24	69.1	38.3	55.0	70.0	48.3	53.3	75.8	41.7	56.5
	Third-y	27	72.6	41.5	57.0	58.5	70.4	50.4	66.7	43.0	57.5
	Forth-y	8	65.7	45.0	55.0	72.5	77.5	60.0	90.0	52.5	64.7
	Total	80	69.8	35.5	49.5	60.0	62.3	48.8	73.3	41.8	55.1

The overall score in mathematical thinking test = 40

Table (2) indicates that the mean of the students' scores on the mathematical thinking test is (55.1). In addition, it shows that the mean of students' scores for the first-year is (46.8), for the second-year it is (56.6), for the third-year, it is (57.5) and for the fourth-year it is (64.7). This clearly indicates that students' mathematical thinking improves as they move from one year to the other. It is noted in table (2) that the mean of students' score from science specialization in the secondary school is (63.8) and it is the best. In addition, the mean of students' scores from Arts specialization is (55.2), for the IT it is (50.2) and for other specializations, it is (58.1). With regard to the eight aspects of mathematical thinking, it is noted in Table (2) that students showed their best performance in the Modeling where the average (73.3). This was followed by Induction with an average of (69.8), Logical Thinking at (62.3), Deduction at (60), Symbolism at (49.5), Estimate at (48.3), Mathematical Proof at (41.8), and Generalization at (35.5).

Second question: Are there differences in the pre-service students' level of math thinking based on specialization in high school?

The researcher calculated the mean and standard deviation values of the students' scores in the different aspects of mathematical thinking, and on the overall test categorized according to their specialization in high school. Table (3) shows these findings.

Table (3).The mean, standard deviation and (F) values of the students' scores in the different aspects of mathematical thinking and on the overall test according to their specialization in high school.

Specialization	Mean/SD	No	Induction	Generalization	Symbolism	Deduction	Logical Thinking	Estimate	Modeling	Proof	Math-Thinking
Sciences	Mean	15	65.3	52.0	62.7	74.7	66.7	53.3	81.3	54.7	63.8
	SD		26.7	36.9	32.0	28.8	24.7	35.2	24.5	22.0	19.0
Art	Mean	31	76.1	30.3	53.5	56.8	65.8	49.0	72.9	37.4	55.2
	SD		29.4	30.1	29.8	28.3	23.2	26.8	28.5	29.1	18.9
IT	Mean	30	64.7	32.0	37.3	56.7	56.7	45.3	68.7	40.0	50.2
	SD		30.9	36.6	32.7	37.5	31.5	30.1	33.5	28.8	24.2
Others	Mean	5	75.0	40.0	60.0	55.0	60.0	55.0	80.0	40.0	58.1
	SD		30.0	32.7	16.3	44.3	36.5	30.0	28.3	28.3	22.9
Total	Mean	80	69.8	35.5	49.5	60.0	62.3	48.8	73.3	41.8	55.1
	SD		29.5	34.4	32.0	33.1	27.4	29.5	29.7	28.0	21.4
F			0.931	1.540	2.781	1.224	0.722	0.308	0.672	1.379	1.410
Sig			0.430	0.211	0.047	0.307	0.542	0.820	0.572	0.256	0.246

Table (3) shows that the mean values of science specialization in the overall test are higher than those of other specialization. To test whether these findings were statistically significant, the researcher calculated the (F) values, which are shown in Table (3). The result indicates that these values were not significant. However, it is noted that there is a significant statistical difference between the mean scores of students in one of the aspects of mathematical thinking, which is Symbolism. To identify the source of this difference, the LSD post Hoc test has been applied. The results of the comparisons indicate that there is significant statistical difference between the mean scores of students of science specialization and IT specialization in favor of students of science specialization.

Third question: Are there differences in the pre-service students' level of math thinking based on the academic year level?

The researcher first calculated the mean and standard deviation values of the students' scores in the different aspects of mathematical thinking and on the overall test, categorized according to their academic year level. Table (4) shows these findings.

Table (4): The Mean, Standard Deviation and (F) Values of the Students' Scores in the Different Aspects of Mathematical Thinking and on the Overall Test According to Their Year Level

Accademic Year_level	Mean/S.D	N	Induction	Generealization	Symbolism	Deduction	Logical thinking	Estimate	Modeling	Math-Proof	Math-Thinking
First_y	Mean	21.00	68.57	20.95	31.43	45.71	61.90	37.14	72.38	36.19	46.79
	S.D		28.69	31.29	27.26	32.95	29.60	27.77	37.14	32.01	20.39
second_y	Mean	24.00	69.17	38.33	55.00	70.00	48.33	53.33	75.83	41.67	56.46
	S.D		31.75	33.84	36.48	33.88	25.65	31.02	27.01	27.61	23.39
Third_y	Mean	27.00	72.59	41.48	57.04	58.52	70.37	50.37	66.67	42.96	57.50
	S.D		28.90	35.49	27.01	30.34	23.12	27.38	27.74	27.01	20.42
Fourth_y	Mean	8.00	65.00	45.00	55.00	72.50	77.50	60.00	90.00	52.50	64.69
	S.D		31.62	35.05	31.62	30.12	24.93	32.07	15.12	21.21	17.39
Total	Mean	80.00	69.75	35.50	49.50	60.00	62.25	48.75	73.25	41.75	55.09
	S.D		29.51	34.42	31.98	33.07	27.37	29.48	29.67	27.96	21.42
F			0.16	1.84	3.32	2.58	4.13	1.74	1.38	0.68	1.79
Sig			0.92	0.15	0.02	0.06	0.01	0.17	0.26	0.57	0.16

It is noted from Table (4) that the students' mean on the mathematical thinking test increased when moving from the first academic year to the second academic year and then to the third and fourth. It is noted from the analysis of variance on the average scores of students in the four academic years that there are no significant statistical differences between students' means in the four academic school years.

However, there was a significant statistical difference on students' means in two aspects of the mathematical thinking, which are Symbolism and Logical Thinking. To identify the sources of these differences, the LSD post Hoc test has been applied. The results indicate that there are significant statistical differences between the mean scores of students of the second academic year and those in the first academic year in favor of students in the second academic year. In addition, there are significant statistical differences between the mean scores of students in the third academic year and those for first academic year students in the symbolism aspect in favor of third academic year students. In addition, there were significant statistical differences between the mean scores of students in the third and fourth year with those of second academic year students in the Logical Thinking aspect in favor of students in the third and fourth academic year.

Fourth question: What are the pre-service students' attitudes towards mathematics in overall inventory and in its four dimensions?

The researcher-classified degrees of the attitudes as follows: Negative if mean scores are between (20-40), Neutral if mean scores are between (40-60) and Positive if mean scores are between (60-100). In addition, the researcher calculated the percentage of students' scores in the inventory as a whole, as well as their percentage of scores for each inventory dimension. Table (5) shows these findings.

Table 5: the Percentage of Students 'Scores in the Inventory as a Whole, As well as Their Percentage of Scores for Each Inventory Dimension

Attitude I	Enjoyment		Motivation		Self Confidence		Value Of Math.		Attitude	
	No	Percent	No	Percent	No	Percent	No	Percent	No	Percent
Negative	2	2.5%	2.0	2.5%	5	6%				
Neutral	20	25%	20.0	25%	20	25%	2	2.5%	17	21%
Positive	58	72.5%	58	72%	55	69%	78	97.5%	36	79%
Total	80	100.0	80.0	100	80	100	80	100	80	100

Table (5) indicates that no students have negative attitude towards mathematics. The percentage of students who have positive attitudes was (79%), and those who have neutral attitudes was (21%). To determine the level of the students' attitude within the four dimensions, the researcher calculated the frequencies and percentages for students' grades within a specific classification. Table (5) shows that the percentage of students who have positive attitudes according to the inventory dimensions in descending order is as follows: Value of Math (97.5%), Enjoyment (72.5%), Motivation (72%) and Self Confidence (69%).

Fifth question: Are there differences in the pre-service students' attitudes towards mathematics based on specialization in high school and academic- year level?

The researcher calculated the mean and standard deviation values of the students' score in the different dimensions of the attitudes toward the mathematics inventory and on the overall inventory categorized according to their specialization in high school and academic year levels. Table (6) shows these findings. Table 6.The mean and standard deviation values of the students' score in the different dimensions of the attitudes toward the mathematics inventory and on the overall inventory categorized according to their specialization in high school and academic year levels

Table 6.The mean and standard deviation values of the students' score in the different dimensions of the attitudes toward the mathematics inventory and on the overall inventory categorized according to their specialization in high school and academic year levels

Specialization	Year	Enjoyment		Motivation		Self-Confidence		Value of Math.		Attitudes	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Science	First-y	83.0	4.2	78.0	8.5	62.0	4.7	95.0	1.4	77.5	0.7
	second-y	87.6	8.9	84.0	11.3	85.9	11.1	92.0	7.6	87.6	5.5
	Third-y	84.5	19.8	86.0	4.0	76.0	12.5	89.5	15.1	82.8	12.0
	Fourth-y	86.5	10.4	68.0	21.4	69.7	15.3	87.0	13.7	78.0	11.6
	Total	85.9	11.6	79.5	14.2	75.7	14.0	90.4	10.6	82.4	9.4
Arts	First-y	59.7	23.1	61.1	22.5	54.9	17.4	74.3	12.0	61.7	14.9
	second-y	72.3	12.8	68.6	10.7	63.2	16.0	91.1	8.2	73.1	9.4
	Third-y	64.6	18.1	60.5	13.8	58.1	15.8	81.9	13.5	66.0	14.2
	Fourth-y	66.0		60.0		62.7		90.0		70.0	
	Total	65.3	18.0	62.5	15.2	58.7	15.7	82.5	13.0	66.8	13.4
IT	First-y	72.2	16.3	69.7	17.7	71.6	15.8	84.0	17.0	74.6	13.7
	second-y	75.7	15.3	78.3	15.0	78.2	12.6	83.3	10.4	78.9	11.3
	Third-y	69.5	23.5	67.0	27.8	59.7	25.5	80.0	15.7	68.1	22.7
	Fourth-y	76.0	33.9	70.0	31.1	74.0	33.0	90.0	14.1	78.0	28.3
	Total	73.5	17.1	72.8	18.4	72.8	17.2	83.6	13.7	75.7	14.6
Others	Third-y	71.3	2.3	61.3	6.1	64.0	6.1	70.0	11.1	67.0	2.2
	Fourth-y	78.0		80.0		66.7		98.0		79.0	
	Total	73.0	3.8	66.0	10.6	64.7	5.2	77.0	16.7	70.0	6.3
Total	First-y	69.0	19.0	67.6	18.8	65.1	17.1	81.8	15.6	70.6	14.6
	second-y	77.2	14.1	76.7	13.9	75.4	15.4	87.4	9.8	79.0	10.8
	Third-y	69.0	18.7	65.3	16.9	61.6	16.8	81.4	14.0	68.9	15.1
	Fourth-y	80.3	16.4	69.0	19.1	69.5	16.4	89.5	11.1	77.1	13.4
	Total	72.6	17.6	69.7	17.2	67.5	17.1	84.1	13.2	73.2	14.1

Table (6) shows that the mean scores of students on the attitudes toward mathematics inventory (73.2), which is classified as positive. In addition, the students' mean values of science specialization in the inventory is the highest (82.4), followed by the IT specialization (75.7), other specialization (70) and finally Arts specialization (66.8). ANOVA test was used to find out if there were significant statistically differences between the mean students' scores according to specialization in high school and the academic year level. Table (7) shows these findings.

Table (7): Means and S.D for the Students' Scores on the Attitudes toward Mathematics Inventory and the (F) Values Based on Specialization

		N	Mean	SD	Min	Max	F	Sig
Enjoyment	Science	15	85.87	11.575	56	100	5.462	0.002
	Arts	31	65.29	17.960	22	90		
	IT	30	73.47	17.059	42	100		
	Others	4	73.00	3.830	70	78		
	Total	80	72.60	17.558	22	100		
Motivation	Science	15	79.47	14.172	40	96	4.335	0.007
	Arts	31	62.45	15.168	20	92		
	IT	30	72.80	18.417	44	100		
	Others	4	66.00	10.583	56	80		
	Total	80	69.70	17.170	20	100		
Self-Confidence	Science	15	75.73	13.959	52	99	5.810	0.001
	Arts	31	58.67	15.682	28	84		
	IT	30	72.80	17.231	33	97		
	Others	4	64.67	5.164	59	71		
	Total	80	67.47	17.102	28	99		
Value of Math	Science	15	90.40	10.642	68	100	1.729	0.168
	Arts	31	82.52	12.992	56	100		
	IT	30	83.60	13.700	42	100		
	Others	4	77.00	16.693	60	98		
	Total	80	84.13	13.222	42	100		
Attitude	Science	15	82.40	9.351	65	94	5.461	0.002
	Arts	31	66.76	13.363	40	89		
	IT	30	75.67	14.617	44	98		
	Others	4	70.00	6.258	65	79		
	Total	80	73.19	14.086	40	98		

It is noted from the analysis of variance on the mean scores of students in the four different specializations on attitudes towards mathematics inventory that there are significant statistical differences between the means of the students as a whole. Furthermore, it shows that there are significant statistical differences in all inventory dimensions except for the value of mathematics dimension. To identify the sources of these differences, the LSD post Hoc test has been applied. The results of the comparisons indicate that the attitudes of students from the science specialization toward math in the overall inventory are better than the attitudes of students from the Arts specialization. In addition, the attitudes of students from IT specialization are better than the attitudes of students in the Arts specialization. Results also indicatethat the attitudes of students from IT specialization toward math are better than the attitudes of students from the Arts specialization in the motivation and self-confidence inventory dimensions. The table also shows that the attitudes of students from the science specialization are better than the attitudes of students in the IT specialization in the enjoyment inventory dimension.

It is noted in table (6) that the mean of the students' scores on mathematical thinking test for the second year is the highest (79), followed by the mean scores for the fourth year (77.1), the mean of the first year (70.6) and finally the mean of the third year (66.9).ANOVA test was used to find out if there were significant statistical differences between the means of students' scores according to the academic year level. Table (8) shows these findings.

Table (8): Means And S.D for the Student's Scores on the Attitudes toward Mathematics Inventory and the (F) Values Based on Academic Year Level

		N	Mean	SD	Min	Max	F	Sig
Enjoyment	First-y	21	69.0	19.0	22	94	1.753	0.163
	Second-y	24	77.2	14.1	48	100		
	Third-y	27	69.0	18.7	30	98		
	Fourth-y	8	80.3	16.4	52	100		
	Total	80	72.6	17.6	22	100		
Motivation	First-y	21	67.6	18.8	20	100	2.090	0.109
	Second-y	24	76.7	13.9	48	100		
	Third-y	27	65.3	16.9	36	100		
	Fourth-y	8	69.0	19.1	40	92		
	Total	80	69.7	17.2	20	100		
Self-Confidence	First-y	21	65.1	17.1	28	97	3.212	0.028
	Second-y	24	75.4	15.4	33	99		
	Third-y	27	61.6	16.8	31	91		
	Fourth-y	8	69.5	16.4	51	97		
	Total	80	67.5	17.1	28	99		
Value of Math	First-y	21	81.8	15.6	42	100	1.564	0.205
	Second-y	24	87.4	9.8	62	100		
	Third-y	27	81.4	14.0	60	100		
	Fourth-y	8	89.5	11.1	68	100		
	Total	80	84.1	13.2	42	100		
Attitude	First-y	21	70.6	14.6	45	97	2.847	0.043
	Second-y	24	79.0	10.8	58	97		
	Third-y	27	68.9	15.1	40	95		
	Fourth-y	8	77.1	13.4	58	98		
	Total	80	73.2	14.1	40	98		

It is noted from the analysis of variances on the means scores of students in the academic year level on the attitudes toward mathematical thinking inventory, that there are significant statistical differences between mean scores of the students as a whole, as well as there are statistical significant differences between mean scores of the students in the self-confidence dimension. To identify the sources of these differences the LSD post Hoc test has been applied. The results of the comparisons indicate that the attitude of second academic year students toward mathematics are better than attitude of first academic year students on the overall test and on the self-confidence dimension. In addition, the attitude toward mathematics of third academic year students are better than the attitude of second academic year students on the overall test, but the attitude toward mathematics of second academic year students are better than the attitude of third academic year students on the self-confidence dimension.

The sixth question: Is there a correlation between the pre-service students' mathematics thinking and their attitudes towards mathematics?

To identify whether a correlation exists between the level of students' mathematical thinking and students' attitudes toward mathematics, the researcher calculated the correlation coefficient between students' attitudes toward mathematics and their ability to think mathematically, the results show a positive correlation coefficient (0.233) which is statistically significant at alpha (0.05).

4. Discussion

This study aims to investigate the mathematical- thinking level of pre-service students and the relationship between students' mathematical thinking and their attitudes towards mathematics. The study shows that student average in the mathematical thinking test was (55%), which is reflects moderate performance, and that the performance of the students was the best in two mathematical thinking aspects: Modeling and Induction. Students' performance was weak in two aspects: Mathematical Proof and Generalization; however, their performance in other aspects was moderate. In light of these results, we can conclude that the mathematics curriculum focuses on the development of certain mathematical- thinking aspects and neglects others.

This is consistent with the results of Nejem's (2004) study, which confirms that mathematics curricula focus on mathematical content and neglects the development of mathematical thinking. In addition, Abu Al-Haija's study (2004), proposes that the reason for the weakness in mathematical thinking can be attributed to the deficiencies in the performance of teachers in developing mathematical- thinking aspects through mathematical activities. In light of this, we can say that the results of this study serve as a sign for curricula planners and teachers in order to develop appropriate plans to improve and develop mathematical thinking aspects among students. The Ministry of Education and the Ministry of higher Education should focus on developing appropriate training programs in mathematical thinking and mathematical strategies for pre-service and in-service teachers. This would reflect positively on students both in schools and hopefully in universities. This finding is consistent with the findings of (Al-Khateeb and Ababneh, 2011: Abed & Asha, 2009; Nejem, 2012; Eleela, 2012; Breen &O'Shea, 2010). Van (2006) confirms that mathematical thinking can be developed through practical and applied contributions.

This study also demonstrates that the performance of students on the mathematical-thinking test reflects improvement in some aspects of mathematical thinking as the students move from one academic year to another. This finding is consistent with Abed & Abu Zainah (2012), who assert that mathematical thinking abilities improve when students move on from high school to university and from one academic year to another. The study also shows that the students' performance in the mathematical-thinking test varies in some aspects depending on students' specialization in secondary school, in favor of students of the Science Specialization. This indicates that the development of mathematical thinking relies heavily on content development, mathematical situations and the experiences related to them. There is a noticeable focus on math and various mathematical skills in the Science Specialization in secondary schools in order to prepare students for science disciplines at university level. These results are confirmed by Katagiri (2004).

This study also shows that students have positive attitudes toward mathematics. They scored (79%) on (ATMI).Students' attitudes are highly positive on the value of mathematics dimension and on the self-confidence dimension, followed by both enjoyment and motivation dimensions.

Although students in Jordan suffer from difficulties in mathematics, as pointed out by Abed & Asha (2009), none of the subjects at the University of Petra had negative attitudes towards mathematics. The researcher noted this fact repeatedly while teaching a course entitled "Basic Concepts in Mathematics". Despite the weakness of achievement in mathematics, students' attitudes were positive. This result is extremely significant as positive attitudes towards mathematics are important in order to gear students toward further learning and choosing a future career in mathematics-related fields. This result is also confirmed by Sanci (2014). Tapia & Marsh (2000)refer to the same idea in their paper. Ernest (1988) however, asserts that mathematical knowledge lies behind attitudes towards mathematics, an idea which is contrary to the belief of many researchers. This study points to the existence of factors other than mathematical knowledge that affect attitudes towards mathematics.

Study results also show that the attitudes of students in the Scientific Specialization at secondary-school level are better than the attitudes of students in other specializations. Students of IT Specialization had better attitudes towards mathematics than those of students specializing in the Arts.

This is a logical finding as students choose which specialization to join depending on their attitudes towards science subjects, particularly mathematics. The study also shows that the attitudes of second academic year students were better than the attitudes of first academic year students. Moreover, the attitudes of the third academic year students were better than the attitudes of second academic year students on the overall test but the attitude toward mathematics of second academic year students are better than attitude of third academic year students on the self-confidence dimension, further investigation is needed to clarify this finding. The study also show a positive correlation between mathematical thinking and attitudes toward mathematics, which needs to be further studies to reveal the nature of this relation.

This study draws the attention of those involved in planning mathematical curricula and programs in various levels of study to the importance of developing students' mathematical thinking to help them change their attitudes to become more positive towards mathematics. Our educational institutions, in their ongoing pursuit of improving the educational process of learning and moving from a conservation and indoctrination culture into the creativity and thinking culture, should work to provide all the educational opportunities that will help develop students' mathematical thinking.

According to Nejem (2012), this can be achieved through either the development of mathematics curriculum and its educational materials or by applying modern teaching and evaluating methods.

5. Recommendations

In light of the results of this study, it is suggested that the following recommendations be taken into consideration:

1- Infusing education courses with materials that deal with enhancing students' abilities in mathematical thinking.
2- Urging educators, mathematics-environment designers and education planners of mathematics curricula to take into consideration the results of this study and the results of other studies to improve math textbooks in different educational levels. In addition, they must develop and design meaningful teaching activities, which focus on improving mathematical thinking skills.
3- Urging mathematics teachers to develop mathematical-thinking skills among students through the teaching and learning of mathematics, which will have a great impact on improving student achievement. There is a need to prepare and qualify math teachers in order to equip them with academic knowledge and professional skills, which, in turn, enables them to develop mathematical thinking among students. This can be done at the pre-service stage or college-preparation phase.
4- Further studies are needed in the areas of mathematical thinking and attitudes toward mathematics and the relationship between the two.

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